Abstracts: Topological Data Analysis – Theory and Applications

May 1-3, 2021

Robert Ghrist (Univ of Pennsylvania)

Title: Laplacians and Network Sheaves

Abstract: This talk will begin with a simple introduction to cellular sheaves as a generalized notion of a network of algebraic objects. With a little bit of geometry, one can often define a Laplacian for such sheaves. The resulting Hodge theory relates the geometry of the Laplacian to the algebraic topology of the sheaf. By using this sheaf Laplacian as a diffusion operator, we will be able to do dynamics on sheaves, which leads to decentralized methods for computing sheaf cohomology. This talk will be grounded in examples arising in applications, with a particular focus on social networks and opinion dynamics, with problems of consensus and polarization as being especially well-suited to sheaf-theoretic analysis. The talk represents joint works with Jakob Hansen and Hans Riess.

Time: Saturday, May 1, 11:00-12:00

Henry Adams (Colorado State)

Title: Bridging applied and quantitative topology

Abstract: I will survey emerging connections between applied topology and quantitative topology. Vietoris-Rips complexes were invented by Vietoris in order to define a (co)homology theory for metric spaces, and by Rips for use in geometric group theory. More recently, they have found applications in computational topology for approximating the shape of a dataset. I will describe a recent result by my student Michael Moy showing that Vietoris-Rips simplicial complexes and Vietoris-Rips metric thickenings (defined using optimal transport) have the same persistence diagrams, and based upon this work I will speculate on the homotopy types of Vietoris-Rips complexes of n-spheres.

Time: Saturday, May 1, 13:00-14:00

Gunnar Carlsson (Stanford/Ayasdi)

Title: Topological Deep Learning

Abstract: Machine learning using neural networks is a very powerful methodology which has demonstrated utility in many different situations. In this talk I will show how work in the mathematical discipline called topological data analysis can be used to (1) lessen the amount of data needed in order to be able to

learn and (2) make the computations more transparent. We will work primarily with image and video data.

Time: Saturday, May 1, 14:30-15:30

Amit Patel (Colorado State)

Title: Edit Distance and Persistence Diagrams Over Lattices

Abstract: We build a functorial pipeline for persistent homology. The input to this pipeline is a filtered simplicial complex indexed by any finite lattice, and the output is a persistence diagram defined as the Möbius inversion of a certain monotone integral function. We adapt the Reeb graph edit distance of Landi et. al. to each of our categories and prove that both functors in our pipeline are 1-Lipschitz making our pipeline stable. Thus this work addresses two longstanding open problems in persistent homology: multiparameter persistent homology and functoriality of persistence diagrams.

This is joint work with my PhD student Alex McCleary.

Time: Saturday, May 1, 16:00-17:00

Jose Perea (Michigan State)

Title: Quasiperiodicity and Persistent Kunneth Theorems.

Abstract: A signal is said to be quasiperiodic if its constitutive frequencies are linearly independent over the rationals. With appropriate parameters, the sliding window embedding of such a function can be shown to be dense in a torus of dimension equal to the number of independent frenquencies. I will show in this talk that understanding the persistent homology of these sets can be reduced to a Kunneth theorem for persistent homology and a bit of harmonic analysis. Needles to say, we will describe the needed persistent Kunneth formulae, as well as applications to time series analysis and computational chemistry.

Time: Sunday, May 2, 11:00-12:00

Maia Fraser (Univ of Ottawa)

Title: TDA within hierarchical learning

Abstract: Topological data analysis is increasingly combined with machine learning techniques, for example prior to deep learning as a form of complexity-reducing feature map. Analogues of deep learning can however also be developed for topological data analysis objects themselves, such as simplicial complexes built from data. We discuss some concrete approaches for doing so and touch on learning theoretic considerations of hierarchical learning in general.

Time: Sunday, May 2, 13:00-14:00

Facundo Mémoli (Ohio State)

Title: The Gromov-Hausdorff distance between spheres.

Abstract: The Gromov-Hausdorff distance is a fundamental tool in Riemanian geometry, and also in applied geometry and topology. Whereas it is often easy to estimate the value of the distance between two given metric spaces, its precise value is rarely easy to determine.

In this talk I will describe results that we have obtained which permit calculating the precise value to the Gromov-Hausdorff between certain pairs of spheres (endowed with their geodesic distance).

Time: Sunday, May 2, 14:30-15:30

Emilie Purvine (Pacific Northwest National Lab)

Title: Homology of Graphs and Hypergraphs

Abstract: Graphs and hypergraphs are typically studied from a combinatorial perspective. A graph being a collection of vertices and pairwise relationships (edges) among the vertices, and a hypergraph capturing multi-way or group-wise relationships (hyperedges) among the vertices. But both of these objects have topological structure in addition to their well-studied combinatorial aspects. Graphs, being inherently pairwise objects, can be considered either as a one dimensional simplicial complex or as a metric space using shortest path distance. Hypergraphs, on the other hand, capture group-wise interactions and thus do not have a simple pairwise metric space that captures the complex structure. While hypergraphs do have an associated simplicial complex, there are multiple hypergraphs consistent with the same simplicial complex. In this talk I will survey some recent results on the homology of both graphs and hypergraphs, including persistent homology of metric graphs and a variety of notions of homology for hypergraphs.

Time: Sunday, May 2, 16:00-17:00

Luis Scoccola (Michigan State)

Title: Approximate and discrete vector bundles

Abstract: Synchronization problems, such as the problem of reconstructing a 3D shape from a set of 2D projections, can often be modeled by principal bundles. Similarly, the application of local PCA to a point cloud concentrated around a manifold approximates the tangent bundle of the manifold. In the first case, the characteristic classes of the bundle provide obstructions to global synchronization, while, in the second case, they provide obstructions to dimensionality reduction.

I will describe joint work with Jose Perea in which we propose several notions of approximate and discrete vector bundle, study the extent to which they determine true vector bundles, and give algorithms for the stable and consistent computation of low-dimensional characteristic classes directly from these combinatorial representations.

Time: Monday, May 3, 13:00-14:00

Michael Lesnick (SUNY/Albany)

Title: ℓ_p-Metrics on Multiparameter Persistence Modules.

Abstract: Motivated both by theoretical and practical considerations in topological data analysis, we generalize the p-Wasserstein distance on barcodes to multi-parameter persistence modules. For each $p \in [1, \infty]$, we in fact introduce two such generalizations $d_{\mathcal{I}}^p$ and $d_{\mathcal{M}}^p$, such that $d_{\mathcal{I}}^\infty$ equals the interleaving distance and $d_{\mathcal{M}}^\infty$ equals the matching distance. These distances turn out to have several good properties. We use them to study the continuity of (2-parameter) multicover persistent homology, revealing nuances to the stability theory for multicover persistence which are not seen by the interleaving distance.

Joint work with Håvard Bjerkevik

Time: Sunday, May 2, 14:30-15:30

John Healy (Tutte Institute)

Title: Practical Clustering and Topological Data Analysis

Abstract: I will give a topologically biased history of useful and popular clustering from a data science perspective with links to the language of topological data analysis. Another way to phrase that could be: useful topological data analysis from the perspective of a data science practitioner. This history will range from non-topologically motivated algorithms such as k-means through the advent of topological methods such as single linkage clustering to more modern techniques such as DBSCAN, HDBSCAN and the place of dimensionality reduction techniques such as UMAP in the problem space of clustering. Throughout the talk I will include citations to practical papers that make use of these algorithms for solving real world problems as well as relevant theory papers for folks to follow up on after the talk.

Time: May 3, 16:00-17:00
