Calculus 2502A Final Examination Monday, December 8, 2008

PART A (30 marks) (2 marks for each problem)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN THIS SECTION MUST BE INDICATED ON THE SCANTRON SHEET. FOR SAFETY, ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

A1. [*2 marks*] Which of the following lines is passing through the origin and is perpendicular to the plane $2x - 5y + z = 6$?

A: $x = 4t, y = 2t, z = -2t$	B: $x = 1 + t, y = 0, z = -2t - 2$
C: $x = -5t, y = 2t, z = t$	D: $x = -5t, y = 2t, z = -t$
E: none of A,B,C,D	

A2. [*2 marks*] Identify the traces of the surface $x^2 - 2y^2 - 4z = 1$ in the indicated planes

A: $x = k$ hyperbolas; $y = k$: circles, $z = k$: hyperbolas
B: $x = k$ parabolas; $y = k$: parabolas, $z = k$: hyperbolas
C: $x = k$ hyperbolas; $y = k$: parabolas, $z = k$: hyperbolas
D: $x = k$ hyperbolas; $y = k$: ellipses, $z = k$: hyperbolas
E: none of A,B,C,D

A3. [*2 marks*] The spherical coordinates (ρ, θ, ϕ) of the point $(0, 1, 1)$ are

A: $(\sqrt{2}, \frac{\pi}{2}, \frac{\pi}{4})$	B: $(\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{2})$	C: $(\sqrt{2}, 0, \frac{\pi}{4})$	D: $(\sqrt{2}, \frac{\pi}{4}, 0)$	E: none of A,B,C,D

A4. [*2 marks*] If $e^{yz} = xy + z^2$ defines z implicitly as a function of x and y, then $\frac{\partial z}{\partial y}=$

A: $\frac{ze^{yz}-x}{ye^{yz}-2z}$	B: $\frac{x-ze^{yz}}{ye^{yz}-2z}$;	C: $\frac{ye^{yz}-2z}{ze^{yz}-x}$;	D: $\frac{ye^{yz}-2z}{x-ze^{yz}}$;	E: none of the above

A5. [*2 marks*] Which of the following is the tangent plane to $\arctan(yz) = \ln(x+z)$ at $(0, 0, 1)$?

A: $x - y + z = 1$	B: $x + y + z = 1$	C: $x - y - z = 1$	D: $-x + y - z = 1$	E: none of A,B,C,D

A6. [*2 marks*] Level curves of the function $f(x, y) = e^{-y/(x^2+1)}$ are

A: ellipses	B: hyperbolas	C: parabolas	D: lines	E: none of A,B,C,D

A7. [*2 marks*] If $v = ye^{xy}$ where $x = s + 2t$ and $y = st$, then when $s = 0$ and $t = 1$, $\partial v/\partial t =$

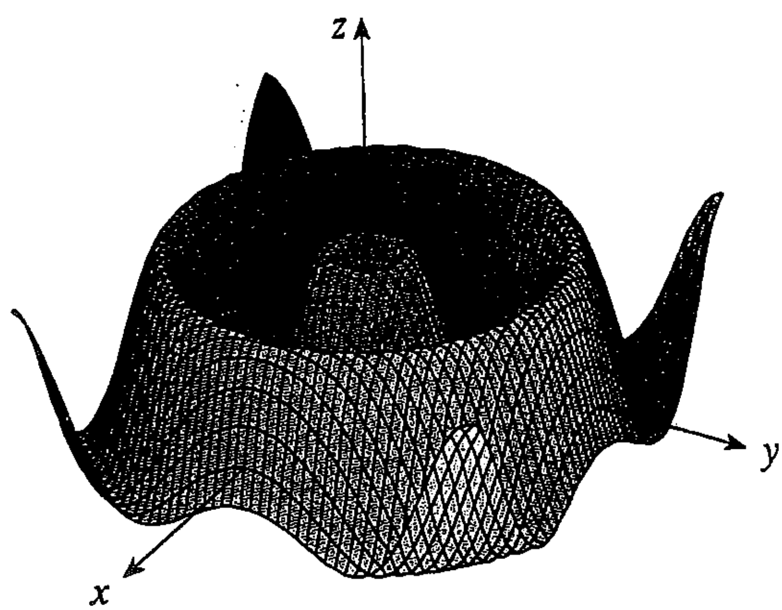
A: 0	B: 1	C: 2	D: -2	E: -1

A8. [*2 marks*] The determinant of the Jacobian matrix for the transformation

$$g(x, y) = (xe^y, xe^{-y})$$

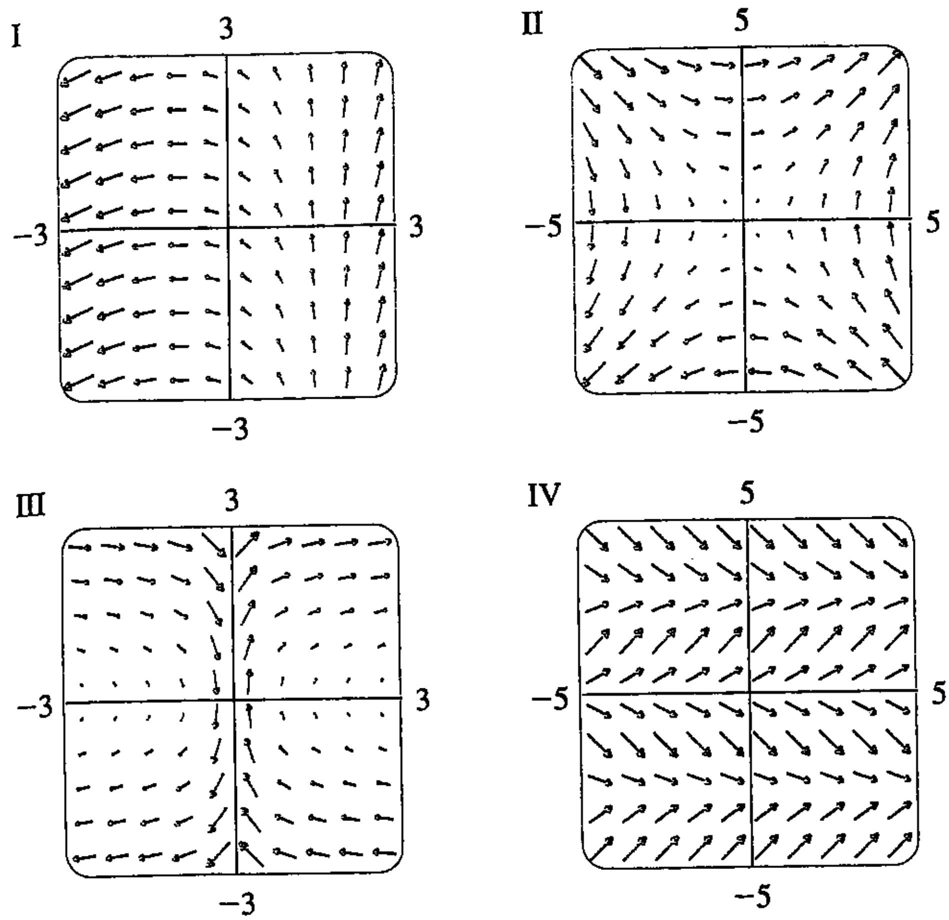
A: $-2xe^y$	B: $2x$	C: $-2x$	D: xe^{2y}	E: none of A,B,C,D

A9. [*2 marks*] Which function has the graph below?



A: $z = \sin(x - y)$	B: $z = \sin(xy)$	C: $z = \sin(x)\sin(y)$	D: $z = \sin(\sqrt{x^2 + y^2})$

A10. [*2 marks*] Which of the vector fields below is the gradient vector field of $f(x,y) = xy$?



A: I	B: II	C: III	D: IV

A11. [*2 marks*] Find the gradient of the scalar field expressed in cylindrical coordinates by

$$f(r,\theta,z) = r\cos(\theta) + z\theta$$

A: $\cos(\theta)\,\mathbf{i} + (z - r\sin\theta)\,\mathbf{j} + \theta\,\mathbf{k}$	B: $\cos(\theta)\,\mathbf{i} + (z/r - \sin\theta)\,\mathbf{j} + \theta\,\mathbf{k}$
C: $\cos(\theta)\,\hat{\mathbf{r}} + (z - r\sin\theta)\,\hat{\boldsymbol{\theta}} + \theta\,\hat{\mathbf{z}}$	D: $\cos(\theta)\,\hat{\mathbf{r}} + (z/r - \sin\theta)\,\hat{\boldsymbol{\theta}} + \theta\,\hat{\mathbf{z}}$
E: none of the above	

A12. [*2 marks*] If $(1,1)$ is a critical point of a function $f(x,y)$ with continuous second derivatives and

$$f_{xx}(1,1) = f_{yy}(1,1) = 3, f_{xy}(1,1) = 2,$$

then at the point $(1,1)$, the function f has

A: a local maximum	B: a local minimum	C: a saddle point	D: none of A,B,C,D

A13. [*2 marks*] If $\mathbf{F} = \langle x, yz, z^2\rangle$ then $(\text{div } \mathbf{F})(\text{curl } \mathbf{F}) =$

A: 0	B: 2F	C: $-(1+3z)y$
D: $\langle -(1+3z)y, 0, 0\rangle$	E: none of the above	

A14. [*2 marks*] If $\mathbf{F} = \langle \cos(5xy), \sin(e^z), x+y+z\rangle$ then div curl $\mathbf{F} =$

A: 0	B: $\langle 0,0,0\rangle$	C: $-25y^2\cos(xy) - e^{2z}\sin(e^z)$
D: $(-25y^2\cos(xy) - e^{2z}\sin(e^z) + 1)^2$	E: none of the above	

A15. [*2 marks*] The Taylor polynomial of degree 4 for

$$f(x,y) = \frac{1}{1-xy}$$

about $(0,0)$ is

A: $1 + (xy) + (xy)^2$	B: $1 - xy + x^2y^2$
C: $1 + (1-xy) + (1-xy)^2$	D: $1 + (1-xy) - (1-xy)^2$
E: none of the above	

PART B (70 marks)

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEMS IN THIS SECTION.

B1. [*8 marks*] Consider two surfaces : S_1: $z = x^2 + y^2$ and S_2: $x^2 + 4y^2 = 4$.

(a) Find a vector equation for the curve of intersection C of S_1 and S_2

(b) Find parametric equations for the tangent line to C at the point $P = (2,0,4)$.

(c) Find the gradient vectors of $f(x,y,z) = x^2 + y^2 - z$ and $g(x,y,z) = x^2 + 4y^2$ at P. Explain how you would use this information to find a direction vector for the tangent line to C at the point P.

B2. [*8 marks*] For the vector function $\mathbf{r}(t) = \langle e^t, e^{-t}, 2t \rangle$, find

(a) the arclength of the curve from $t = 0$ to $t = 1$.

(b) the equation of the normal plane at $\mathbf{r}(0)$.

(c) the equation of the osculating plane at $\mathbf{r}(0)$.

B3. [*6 marks*]

(a) Show that $\lim_{(x,y)\to(0,0)} \frac{xy(x^2-y^2)}{(x^2+y^2)^2}$ does not exist.

(b) Find $\lim_{(x,y)\to(0,0)} \frac{x^2 \sin(y)}{\sqrt{x^2+y^2}}$. Justify your answer.

B4. [*6 marks*] Let $f(x, y) = x^2 - y$.

(a) Make a careful, neat drawing of the level curve of f that goes through $(1, 0)$ and the vector grad $f(1, 0)$ with its initial point at $(1, 0)$. Clearly indicate any important features in your drawing.

(b) Find the rate of change of f at the point $P = (1, 0)$ in the direction from the point P to the point $Q = (0, 1)$.

(c) Find the maximum rate of change of f at $(1, 0)$ and the direction in which it occurs.

B5. [*12 marks*] Use Lagrange multipliers to find the point on the surface $x^2 + y^2 - z^2 = 20$ that is closest to the point $(2, 1, 0)$.

B6. [*12 marks*] Find the absolute maximum and minimum values and any saddle points of the function $f(x, y) = x^2 - xy - x + y$ on the triangular region D whose vertices are $(0, 0)$, $(2, 0)$ and $(2, 4)$.

B7. [*6 marks*] Determine whether or not the vector field

$$\mathbf{F}(x, y, z) = \langle e^x \cos(y) + yz, xz - e^x \sin(y), xy \rangle$$

is conservative. If it is, find a function f such that $\mathbf{F} = \nabla f$. In either case, explain carefully.

B8. [*6 marks*] Find the Jacobian matrix $D\mathbf{f}(x, y, z)$ for the transformation from $\mathbf{R}^3$ to $\mathbf{R}^2$ given by

$$\mathbf{f}(x, y, z) = (\arctan(x + 2y - 3z), \ln(xyz))$$

Use $D\mathbf{f}(1, 1, 1)$ to help you find an approximate value for $\mathbf{f}(1.1, 0.9, 1.2)$.

B9. [*6 marks*] Find the degree 2 Taylor polynomial of $f(x, y) = \frac{\cos(x)}{y}$ at the point $(0, 1)$.