Calculus 250a Fall 2007 Midterm exam

Tuesday, October 23, 2007; 7:00-9:30 pm

Problem 1. (6 marks)

Let $\vec{x} = \langle 2, 2, -1 \rangle$ and $\vec{y} = \langle 0, 1, 1 \rangle$.

(a) (2 marks) Find $\vec{x} \cdot (2\vec{x} - 3\vec{y})$.

(b) (2 marks) Find $\vec{x} \times \vec{y}$.

(c) (2 marks) Compute $\sin \Theta$, where Θ is the angle between $\vec{x}$ and $\vec{y}$.

Problem 2. (6 marks)

Find parametric equations of the line of intersection of the planes $x - y - z = 1$ and $2x + y + 2z = 2$.

Does this line intersect the line given by parametric equations $x = h + 1,\ y = 2h,\ z = h$?

Problem 3. (5 marks)

Find an equation of the plane passing through the points $(2, 0, 0)$, $(0, 1, 0)$, $(0, 0, -5)$.

Problem 4. (3 marks)

Sketch the surface given by the equation

$$z^2 - 2z = x^2 + y^2 - 2y.$$

Problem 5. (3 marks)

Find the length of the curve

$$\vec{r}(t) = \langle \sin t, \sqrt{2} \cos t, -\sin t \rangle,\ 0 \leq t \leq \frac{\pi}{2}.$$

Problem 6. (4 marks)

Find parametric equations for the tangent line to the curve

$$x = t - \pi,\ y = e^{\sin t},\ z = \cos t$$

at $(0, 1, -1)$.

Problem 7. (5 marks)

Find the position vector $\vec{r}(t)$ of a particle that has acceleration $\vec{a}(t) = \langle 6t, \cos t, e^{-t} \rangle$, initial velocity $\vec{v}(0) = \langle 2, 0, 1 \rangle$ and initial position $\vec{r}(0) = \langle 5, 3, 0 \rangle$.

Problem 8. (1 mark)

The angle of intersection (at $t = 0$) of the curves $\vec{r}(t) = \langle \cos t, \sin t, t + 1 \rangle$ and $\vec{q}(t) = \langle 1 - t^3, t^2 - t, e^t \rangle$ is

(A) π
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{4}$
(D) 0
(E) none of the above

Problem 9. (1 mark)

A unit vector perpendicular to both $\vec{a} = \vec{i} + 3\vec{k}$ and $\vec{b} = 3\vec{i} + \vec{j}$ is

(A) $-3\vec{i} + \vec{k}$
(B) $-3\vec{i} + 9\vec{j}$
(C) $-3\vec{i} + 9\vec{j} - \vec{k}$
(D) $-3\vec{i} + 9\vec{j} + \vec{k}$
(E) none of the above

Problem 10. (1 mark)

The surface which consists of all points P for which the distance from P to the z-axis is twice the distance from P to the xy-plane is

(A) a cone
(B) an ellipsoid
(C) an elliptic paraboloid
(D) a plane
(E) none of the above.