Calculus 2502A Midterm Examination Thursday, October 23, 2008

PART A (20 marks) (2 marks for each problem)

NOTE: YOUR ANSWERS TO THE PROBLEMS IN THIS SECTION MUST BE INDICATED ON THE SCANTRON SHEET. FOR SAFETY, ALSO CIRCLE YOUR ANSWERS IN THIS BOOKLET.

A1. [*2 marks*] The distance from the point $(2, -5, 1)$ to the y-axis is

A: $\sqrt{30}$	B: $\sqrt{29}$	C: $\sqrt{26}$	D: $\sqrt{5}$	E: none of A,B,C,D

A2. [*2 marks*] If $\mathbf{u}$ is a unit vector, then the length of the vector $-3\mathbf{u}$ is

A: 3	B: $\frac{1}{3}$	C: $\sqrt{3}$	D: $\frac{1}{\sqrt{3}}$	E: none of A,B,C,D

A3. [*2 marks*] The vector projection $\mathbf{proj_a b}$ of $\mathbf{b} = \langle 1, 1, 2\rangle$ onto $\mathbf{a} = \langle -3, 0, 1\rangle$ is

A: $\frac{1}{\sqrt{10}}$	B: $-\frac{1}{\sqrt{10}}$	C: $\langle \frac{3}{10}, 0, -\frac{1}{10}\rangle$	D: $\langle -\frac{3}{10}, 0, \frac{1}{10}\rangle$	E: none of A,B,C,D

A4. [*2 marks*] Lines

$$L_1: \quad x = 2 - t, \; y = 3t, \; z = 5 + 8t$$
$$L_2: \quad \frac{x+1}{3} = \frac{y+2}{8} = z - 1$$

are

A: parallel	B: perpendicular	C: neither

A5. [*2 marks*] Planes $6x - 2y + 5z = 0$ and $x + 3y + 8 = 0$ are

A: parallel	B: perpendicular	C: neither

A6. [*2 marks*] The domain of $\mathbf{r}(t) = \langle \sqrt{2-t}, (e^t - 1)/t, \ln(t+1)\rangle$ is the interval

A: $(-1, 2]$	B: $[-2, 0) \cup (0, 1)$	C: $[-1, 0) \cup (0, 2)$	D: $(-1, 0) \cup (0, 2]$	E: none of A,B,C,D

A7. [*2 marks*] The curve $\mathbf{r}(t) = \langle \cos(t), \sin(t), \ln(t)\rangle$ is which of the curves in the graphs below:

A: VI	B: II	C: III	D: IV	E: V

A8. [*2 marks*] The curve $\mathbf{r}(t) = \langle t, t^2, e^{-t}\rangle$ is which of the curves in the graphs below:

A: I	B: II	C: III	D: IV	E: V

A9. [*2 marks*] In spherical coordinates, $\rho = \sin(\theta)\sin(\phi)$ represents

A: a sphere	B: a half cone	C: an ellipsoid	D: a paraboloid	E: none of A,B,C,D

A10. [*2 marks*] In cylindrical coordinates, $z^2 = 4 + r^2$ represents

A: a cone	B: a hyperbolic paraboloid	C: a cylinder
D: a hyperboloid of two sheets	E: a hyperboloid of one sheet.	

PART B (80 marks)

NOTE: SHOW ALL YOUR WORK FOR THE PROBLEMS IN THIS SECTION.

B1. [*6 marks*] Let $\mathbf{a} = \langle 2, 3, 1\rangle$, $\mathbf{b} = \langle -2, 1, 1\rangle$, and let Θ be the angle between these vectors. Find the following:

(a) $3\mathbf{a} - 2\mathbf{b} =$

(b) $|\mathbf{b}| =$

(c) $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) =$

(d) $\mathbf{a} \times \mathbf{b} =$

(e) $\cos\Theta =$

(f) the area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$ =

B2. (a) [*3 marks*] Find parametric and symmetric equations for the line through points $(1, 2, 3)$ and $(4, -1, 5)$.

(b) [*3 marks*] Find an equation of the plane passing through the origin and orthogonal to this line.

B3. Let S be the region that lies inside (or on) the cone $z^2 = x^2 + y^2$ and between the xy plane and the plane $z = 2$. Describe S in terms of inequalities using

(a) [*4 marks*] rectangular Cartesian coordinates;

(b) [*4 marks*] cylindrical coordinates;

(c) [*4 marks*] spherical coordinates.

B4. [*12 marks*] For each of the following equations, state

(i) which of the graphs in Figure II from the extra page has the given equation,

(ii) whether w denotes x, y, or z (here w labels an axis in the diagram).

(iii) the correct name of the surface.

(a) $4x^2 - y^2 + z^2 = 4$

(*i*) Graph=

(*ii*) $w =$

(*iii*) Surface=

(b) $-x^2 + 4y^2 + z^2 = 0$

(*i*) Graph=

(*ii*) $w =$

(*iii*) Surface=

(c) $x^2 - z^2 = 1$

(*i*) Graph=

(*ii*) $w =$

(*iii*) Surface=

(d) $x^2 - y - z^2 = 0$

(*i*) Graph=

(*ii*) $w =$

(*iii*) Surface=

B5. [*4 marks*] Let C be the curve of intersection of the hyperboloid $x^2 + y^2 - z^2 = 2$ and the plane $z = x + y$. Find a parametrization of the curve C. Give a description in words of this curve C.

B6. [*6 marks*] For the curve C defined by $\mathbf{r}(t) = \langle 3\sin(t), 5\cos(t), 4\sin(t)\rangle$
(a) find the arclength of the curve as t varies from 0 to 1.
(b) reparametrize the curve with respect to arclength measured from the point where $t = 0$ in the direction of increasing t.

B7. (a) [*4 marks*] Find the parametric equations for the tangent line to the curve C defined by

$$\mathbf{r}_1(t) = \langle te^{-t}, 2\arctan(t), 2e^t\rangle$$

at the point $\mathbf{r}_1(0)$.

(b) [*4 marks*] The curve C given by

$$\mathbf{r}_1(t) = \langle te^{-t}, 2\arctan(t), 2e^t\rangle$$

intersects the curve D given by

$$\mathbf{r}_2(t) = \langle t\cos(t), t\sin(t), 1 + \cos^2(t)\rangle$$

at the point $(0, 0, 2)$. Find the cosine of the angle of intersection of these curves at $(0, 0, 2)$.

B8. [*12 marks*] For the vector function $\mathbf{r}(t) = (\cos(t), e^t, \sin(t))$, find
(a) $\mathbf{r}'(t)$,
(b) $\mathbf{r}''(t)$,
(c) $\mathbf{T}(0)$,
(d) $\kappa(0)$,
(e) $a_T(0)$,
(f) $a_N(0)$.

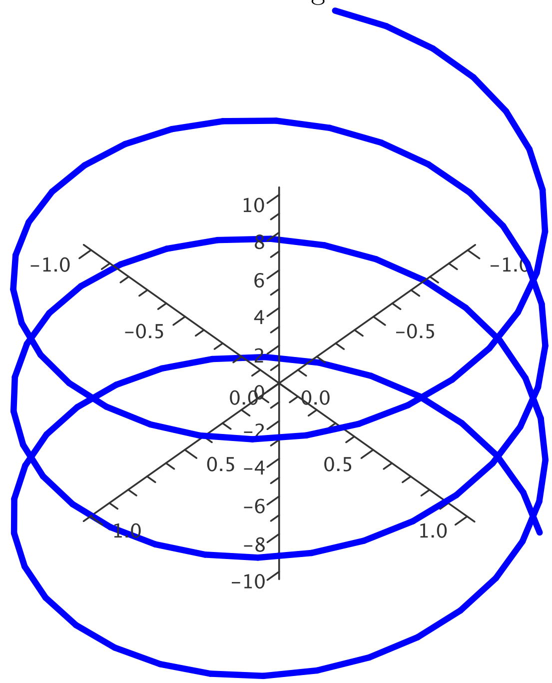
B9. [*10 marks*] For the vector function $\mathbf{r}(t) = \langle \ln(\cos(t)), \sin(t), \cos(t)\rangle$ find
(a) the equation of the osculating plane at $\mathbf{r}(0)$.
(b) the equation of the normal plane at $\mathbf{r}(0)$.

B10. [*4 marks*] Find the velocity and position vectors of a particle that has acceleration given by

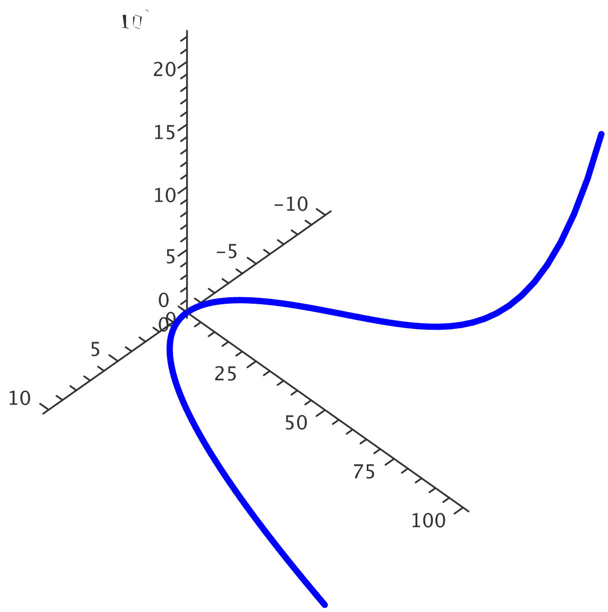
$$\mathbf{a}(t) = \langle 2, e^t, (2+t)e^t \rangle,$$

with initial velocity $\mathbf{v}(0) = \langle 0, 1, 1 \rangle$ and initial position $\mathbf{r}(0) = \langle 0, 1, 0 \rangle$.

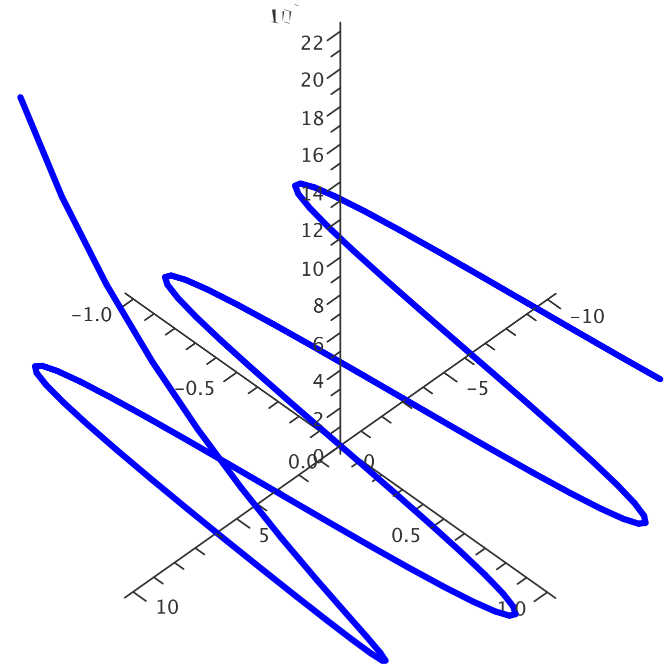
Figure I.



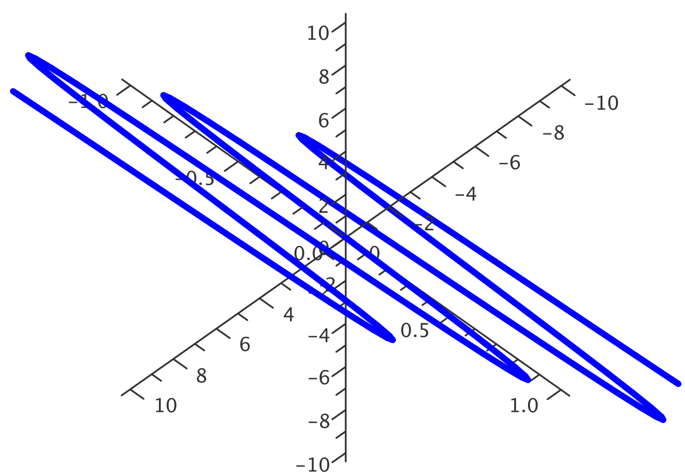
Graph I.



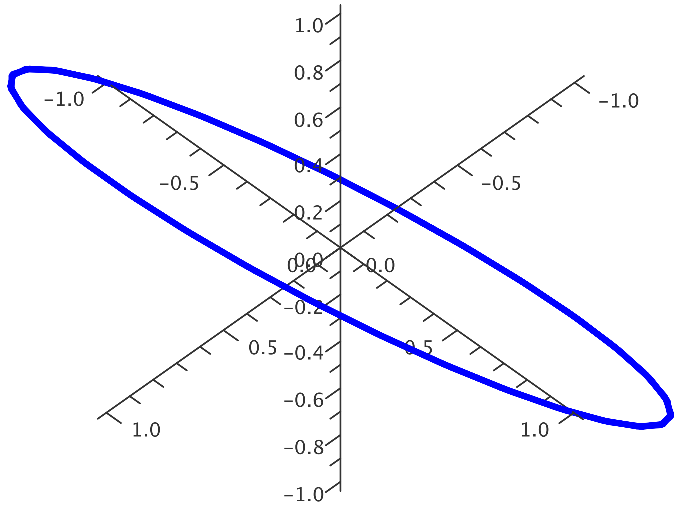
Graph II.



Graph III.

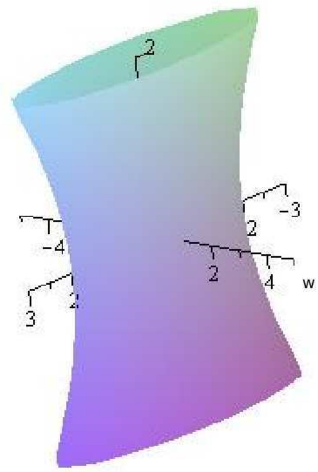


Graph IV.

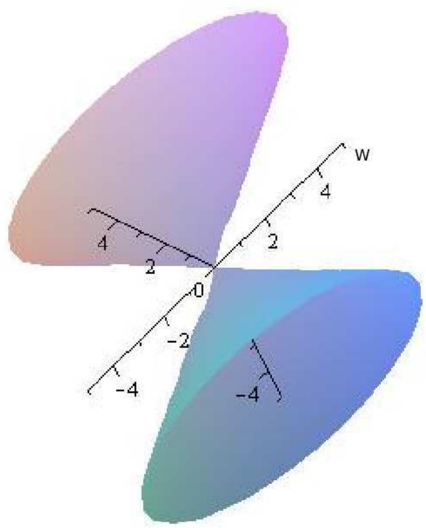


Graph V.

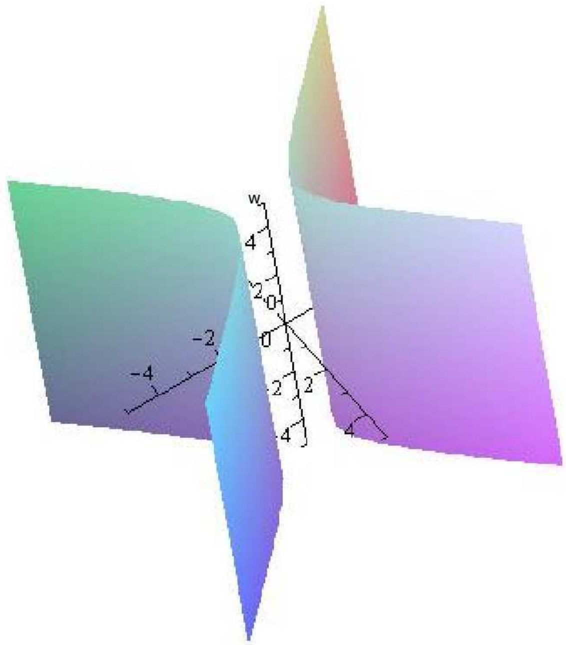
Figure II.



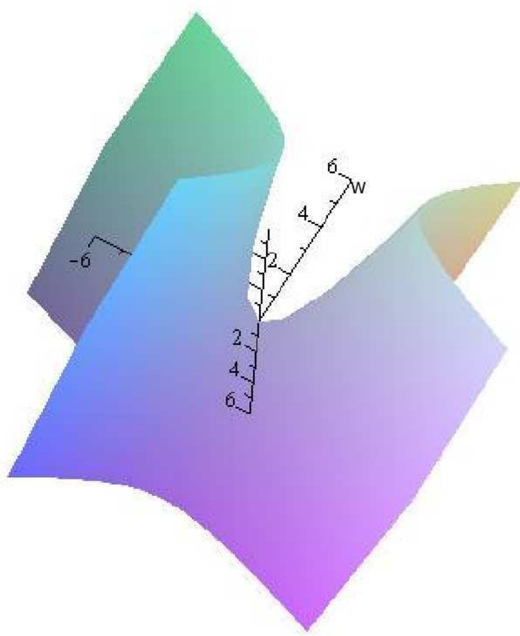
Graph 1.



Graph 2.



Graph 3.



Graph 4.