Homework 1.

Due October 13.

1. Let $M = \{z = (z_1, \ldots, z_n) \in \mathbb{C}^n : x_1 = 0\}$. Suppose that $f(z)$ is a holomorphic function in $\mathbb{C}^n$, such that $f|_M = 0$. Prove that $f \equiv 0$.
2. Problem E.1.9 (Textbook, p. 18)
3. A *complex line* in $\mathbb{C}^n$ is a subset of the form
$$\{z \in \mathbb{C}^n : z = A\zeta + B,\ \zeta \in \mathbb{C}\}$$
for some $A, B \in \mathbb{C}^n$. Prove that two different complex lines in $\mathbb{C}^2$ can intersect at most at one point.
4. Prove that if $f \in \mathcal{O}(\mathbb{C}^n)$, and $|f(z)| \leq C|z^\alpha|$, for some $\alpha \in \mathbb{N}^n$, then $f(z)$ is a polynomial of degree at most $|\alpha|$.