CALCULUS 2302 FALL 2012

PRACTICE QUESTIONS FOR THE FINAL

Note: problems are taken from the previous finals.

7.1. Consider the function $f(x, y) = -4 + (x-1)^2 + (y-1)^2$ on the triangular region where $x \geq 0$, $y \geq 0$, and $y \leq 4 - x$. Find the absolute maximum and absolute minimum values of f on this region, and all points at which they occur.

7.2. Use Lagrange multipliers to find the maximum and minimum values of the function

$$f(x, y, z) = 2x + 6y + 10z$$

subject to the constraint $x^2 + y^2 + z^2 = 35$. List the points at which the maximum and minimum occur.

7.3. Let $M = we^{uv}$, where $u = r + s$, $v = r - s$, and $w = rs$. Using the chain rule, find $\frac{\partial M}{\partial s}$ for $r = 3$ and $s = 2$.

7.4. Find and classify the critical points of $f(x, y) = x^4 + y^4 - 4xy + 2$.

7.5. The temperature at a point (x, y, z) is given by

$$T(x, y, z) = \sqrt{x^2 + 2y^2 + 3z^2}.$$

(a) Find the rate of change of the temperature at the point $P(1, 0, 1)$ in the direction toward the point $Q(3, 1, 3)$.

(b) Give a vector pointing in the direction that the temperature decreases fastest at $P(1, 0, 1)$

(c) What is the directional derivative of T at P in the direction you found in (b)?

7.6. Find the linear approximation of the function

$$f(x, y) = \ln(x^2 + y)$$

at $(0, 1)$ and use it to approximate $f(0.1, 0.9)$.

7.7. Verify that the function $u(x, y) = \ln(x^2 + y^2)$, where $(x, y) \neq (0, 0)$, satisfies the Laplace equation $u_{xx} + u_{yy} = 0$.

7.8. Find the position vector $\vec{r}(t)$ of a particle whose acceleration, initial position and velocity are given by

$$\vec{a}(t) = 6t\vec{i} + e^t\vec{j} + \cos t\vec{k},\ \ \vec{v}(0) = \vec{i} + \vec{j},\ \ \vec{r}(0) = \vec{j} - \vec{k}.$$

7.9. Let $\vec{r}(t) = \langle t^2, \sin t - t\cos t, \cos t + t\sin t\rangle$.

(a) Find parametric equations of the tangent line to $\vec{r}$ at $t = \pi$.

(b) Find the equation of the plane through the point $\vec{r}(\pi)$ which is perpendicular to the curve $\vec{r}(t)$. Write your answer in the form $ax + by + cz + d = 0$.

(c) Find the arc length of $\vec{r}(t)$ for $0 \leq t \leq \pi$.

7.10. Find a tangent plane to the surface

$$x^2 + 3y^2 + z^2 = 8$$

at $(1, -1, 2)$.

7.11. Given that $yz = e^{x+z}$ implicitly defines z as a function of x and y, find the partial derivative $\frac{\partial z}{\partial y}$.

7.12. Find the line that passes through the origin and is perpendicular to the plane $2x-5y+z = 6$.

7.13. Find the tangent plane to $\arctan(yz) = \ln(x + z)$ at $(0, 0, 1)$.

7.14. Consider two surfaces $S_1 : z = x^2 + y^2$ and $S_2 : x^2 + 4y^2 = 4$.
   (a) Find a vector equation for the curve of intersection C of S_1 and S_2.
   (b) Find parametric equations of the tangent line to C at $P = (2, 0, 4)$.
   (c) Find the gradient vectors of $f(x, y, z) = x^2 + y^2 - z$ and $g(x, y, z) = x^2 + 4y^2$ at P. Explain how you would use this information to find a direction vector for the tangent line to C at P.

7.15. Let $f(x, y) = x^2 - y$.
   (a) Make a careful neat drawing of the level curve of f that goes through $(1, 0)$ and the vector $\nabla f(1, 0)$ with its initial point at $(1, 0)$. Clearly indicate any important features in your drawing.
   (b) Find the rate of change of f at the point $P = (1, 0)$ in the direction from the point P to the point $Q = (0, 1)$.
   (c) Find the maximum rate of change of f at $(1, 0)$ and the direction in which it occurs.

7.16. Use Lagrange multipliers to find the point on the surface $x^2 + y^2 - z^2 = 20$ that is closest to the point $(2, 1, 0)$.