CALCULUS 2302 FALL 2012

HOMEWORK ASSIGNMENT 2.

Due October 3.

1.1. Find the acute angle between two lines in $\mathbb{R}^2$ given by

$$x + 2y = 7$$
$$5x - y = 2$$

1.2. (i) Given the vector $\vec{u} = 3\vec{i} + 5\vec{j} - 3\vec{k}$, find a scalar c such that the scalar projection of $\vec{u}$ onto the vector $\vec{v} = -\vec{i} + 2\vec{j} + c\vec{k}$ has length 2.

(ii) Find the vector projection of the vector $\vec{u}$ above onto each $\vec{i}$, $\vec{j}$, and $\vec{k}$.

1.3. Find a unit vector orthogonal to the plane passing through the points $(-1, 3, 1)$, $(0, 5, 2)$, and $(4, 3, -1)$.

1.4. Let

$$\vec{u} = \langle 3, -2, 4\rangle$$
$$\vec{v} = \langle 1, -1, 0\rangle$$
$$\vec{w} = \langle -2, 0, c\rangle$$

(i) Find the constant c such that the volume of the parallelepiped determined by the vectors equals one.

(ii) Find c such that the three vectors are coplanar.

1.5. Given three vectors $\vec{a} = \langle 2, -1, 0\rangle$, $\vec{b} = \langle 3, 1, 2\rangle$, and $\vec{c} = \langle 1, 0, -7\rangle$, find two of them that have the cross product of largest length.

1.6. Let $\mathbf{V}$ be a parallelepiped in $\mathbb{R}^3$ with one vertex at the origin, which is spanned by vectors $a\vec{i}$, $b\vec{j}$, and $c\vec{k}$. Suppose that the directional angles of the biggest diagonal satisfy

$$(\cos\alpha, \cos\beta, \cos\gamma) = \left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right).$$

If the volume of $\mathbf{V}$ equals 5, find a, b, and c.

1.7. Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}$$

1.8. Prove that if vectors $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal, then the vectors $\vec{u}$ and $\vec{v}$ must have the same length.