CALCULUS 2302 FALL 2012

HOMEWORK ASSIGNMENT 4.

Due November 5.

4.1. Find a vector function that represents the curve of intersection of the surfaces $z = 4x^2 + y^2$ and $z = xy$.

4.2. Prove the Chain Rule for vector-valued functions:

$$\frac{d}{dt}\left[\vec{u}(f(t))\right] = f'(t)\,\vec{u}'(f(t)).$$

4.3. Find parametric equations for the tangent line to the curve given by

$$x = \sqrt{t^2+3}$$
$$y = \ln(t^2+1)$$
$$z = 5t$$

at the point $(2, \ln 2, 5)$.

4.4. Use Theorem 10 of the textbook (p. 880) to show that

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{|\dot{x}^2 + \dot{y}^2|^{3/2}}.$$

4.5. Consider the curve $y = x^2$ in the plane. What is its maximal curvature, and at which point(s) does it occur?

4.6. Find the value of constant c such that the curve $\vec{r}(t) = \langle t^2, \ln t, c\,t\rangle$ has curvature 0 at the point $t = 1$.

4.7. Find the velocity acceleration and speed of a particle with the position function

$$\vec{r}(t) = \langle e^{\ln(t^2+1)}, t^{\sin t}, \sqrt{t}\rangle$$

at time $t = 1$.

4.8. Evaluate the integral

$$\int_0^1 \left((te^t)\,\vec{i} + \frac{1}{\sqrt{1-t^2}}\vec{j} + \frac{1}{\cos t}\vec{k} \right) dt.$$

4.9. Reparametrize the curve

$$\vec{r}(t) = 2\,\vec{i} + e^{2t}\cos 2t\,\vec{j} + e^{2t}\sin 2t\,\vec{k}$$

with respect to arc length measured from the point $t = 0$ in the direction of increasing t.