CALCULUS 2302 FALL 2012

HOMEWORK ASSIGNMENT 6.

Due December 3.

6.1. If $u = f(x, y)$, where $x = e^s \cos t$ and $y = e^s \sin t$, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2s}\left(\frac{\partial^2 u}{\partial s^2} + \frac{\partial^2 u}{\partial t^2}\right).$$

6.2. Find the directions in which the directional derivative of $f(x, y) = ye^{-xy}$ at the point $(0, 2)$ has the value 1.

6.3. Consider the function $G(x, y, z) = \sin(x^2+z)-\sin(z^2-y)$. Assume that near $x = y = z = 0$ we define $g(x, y)$ implicitly by $G(x, y, g(x, y)) = 0$. Find $\frac{\partial g}{\partial y}(0, 0)$.

6.4. Compute the tangent plane to the ellipsoid

$$\frac{x^2}{3} + \frac{y^2}{6} + \frac{z^2}{9} = 1$$

at the point $P(-1, \sqrt{2}, \sqrt{3})$.

6.5. Let $f(x, y) = -x^3 + 4xy - 2y^2 + 1$. Find all the critical points of f. For each critical point of f, determine whether it is a local maximum, a local minimum or a saddle point.

6.6. Find the absolute minimum and maximum of $f(x, y, z) = xyz$ in the region $x^2+y^2+2z^2 \leq 3$.

6.7. At what point on the paraboloid $y = x^2 + z^2$ is the tangent plane parallel to the plane $x + 2y + 3z = 1$?

6.8. Find three positive numbers whose sum is 12 and the sum of whose squares is as small as possible.

6.9. Find the maximum and minimum volumes of a rectangular box whose surface area is 1500 cm^2 and whose total edge length is 200 cm.