Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Marks:	4	6	14	6	4	6	3	6	12	4	5	70
Score:												

Name (print): _____

Signature: _____

UWO ID number: _____

The UNIVERSITY of WESTERN ONTARIO
DEPARTMENT of MATHEMATICS

INTERMEDIATE CALCULUS I 2302A MIDTERM EXAMINATION
1 November 2011 6:30-9:30 PM

INSTRUCTIONS:

1. This exam is 12 pages long. It is printed double-sided. There are 11 questions.
2. All questions must be answered in the space provided. Indicate your answer clearly. Should you need extra space, a blank page is provided at the end of the booklet.
3. Show all your of your work and explain your answers fully. Unjustified, irrelevant or illegible answers will receive little or no credit.
4. Do not unstaple the exam booklet.
5. **No aids are permitted. In particular, calculators, cell phones, ipods etc. are not allowed and may be confiscated.**

1. (4 marks) Show that the equation

$$x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$$

represents a sphere, and find its centre and radius.

2. (a) (3 marks) Find the area of the triangle with vertices $P(1, -2, 3)$, $Q(0, 3, 1)$ and $R(-1, 1, 0)$.

(b) (3 marks) Find the volume of the parallelepiped spanned by the vectors $\mathbf{a} = \langle 1, 2, 3 \rangle$, $\mathbf{b} = \langle 0, 1, -1 \rangle$ and $\mathbf{c} = \mathbf{i} + \mathbf{j}$.

3. (a) (4 marks) Let $\mathbf{v}$ be a vector in the third quadrant of the plane that has length $\sqrt{3}$ and makes an angle of $\pi/3$ with the negative x-axis. Write $\mathbf{v}$ in component form.

(b) (4 marks) Let $\mathbf{v}$ and $\mathbf{w}$ be two vectors such that $\mathbf{v} \cdot \mathbf{w} = 1$ and $\mathbf{v} \times \mathbf{w} = \langle 1, 1, 1 \rangle$. Find the angle α between $\mathbf{v}$ and $\mathbf{w}$. (Hint: Try to find $\tan \alpha$ first.)

(c) (6 marks) Let the vector **a** point from the origin to the point on the unit sphere with spherical coordinates $\theta = 270°$ and $\phi = 45°$, and let **b** point similarly to the point with spherical coordinates $\theta = 180°$ and $\phi = 135°$. (Notation as in the textbook.) Find the angle α between **a** and **b**.

4. (a) (3 marks) Find vector equation and parametric equations for the line through the point $(2, 1, -3)$ parallel to the vector $\mathbf{j} - 2\mathbf{k}$.

(b) (3 marks) Find an equation of a plane parallel to the xz-plane passing through the point $P(3, 4, -1)$.

5. (4 marks) Decide whether the following two lines are identical, parallel, skew or intersecting. In the latter case, compute the point of intersection as well.

$$L_1 : \mathbf{r} = 2\mathbf{i} + t(\mathbf{i} - 3\mathbf{k}),$$
$$L_2 : \mathbf{r} = \mathbf{k} + t(\mathbf{j} + \mathbf{k}).$$

6. (a) (4 marks) Compute the line of intersection of the two planes

$$P_1 : x + z = 4,$$
$$P_2 : y - x = 1.$$

(b) (2 marks) At which angle do the two planes intersect?

7. (3 marks) Which kind of quadric is given by the equation $y - x^2 = z^2/2$?

8. (a) (3 marks) Compute the tangent line to the helix $\mathbf{v}(t) = \langle \cos t, \sin t, t \rangle$ at $t = 2\pi/3$.

(b) (3 marks) Find a point on the helix where the tangent vector is parallel to the plan $x + y - \sqrt{2}z = 0$.

9. (a) (3 marks) Compute the unit tangent vector $\mathbf{T}(t)$ for the curve $\mathbf{v}(t) = \langle t, e^t, \cos(2t)\rangle$.

(b) (4 marks) Compute the curvature of $\mathbf{v}(t)$ at $t = 0$.

(c) (5 marks) Compute the length of the arc given by $\mathbf{w}(t) = \langle t^2, 0, t^3 \rangle$ for $0 \leq t \leq 1$.

10. (4 marks) Consider a particle with mass $m = 1$ on which the time-dependent force $\mathbf{F}(t) = e^t\mathbf{j} - t\mathbf{k}$ acts. Find the position vector $\mathbf{r}(t)$, given that the initial position is $\mathbf{r}(0) = \mathbf{i} + 2\mathbf{j}$ and the initial velocity $\mathbf{v}(0) = -\mathbf{i}$.

11. (5 marks) Consider the function $f(x, y) = \ln(y^2 - x)$. Sketch the domain of $f(x, y)$ in the plane as well as the level curve for the level 0.

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