MATH 4155/9055 FALL 2013

HOMEWORK ASSIGNMENT 1. DUE SEPTEMBER 27.

1.1. Derive Minkowski's inequality

$$\left(\sum_{i=1}^{n}(a_i+b_i)^k\right)^{1/k} \le \left(\sum_{i=1}^{n} a_i^k\right)^{1/k} + \left(\sum_{i=1}^{n} b_i^k\right)^{1/k}, \quad a_i>0,\ b_i>0,\ k>1$$

from Hölder's inequality

$$\sum_{i=1}^{n} a_i b_i \le \left(\sum_{i=1} a_i^p\right)^{1/p} \cdot \left(\sum_{i=1} b_i^q\right)^{1/q}, \quad a_i>0,\ b_i>0,\ p,q>1,\ 1/p+1/q=1.$$

1.2. Prove that if a map $f:\mathbb{R}^n \to \mathbb{R}^m$ is differentiable at a point p, then it is continuous there. Give example of a function that is continuous at the origin, has there partial derivatives, but is not differentiable.

1.3. Suppose that a differentiable function $f:\Omega \to \mathbb{R}$ on a domain Ω satisfies $Df(x)=0$ for all $x \in \Omega$. Prove that f is a constant function.

1.4. Consider the function

$$f(x,y) = \begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & (x,y) \ne (0,0) \\ 0, \quad (x,y)=(0,0) \end{cases}$$

Show that $D_{xy}f \ne D_{yx}f$.

1.5. If f and g are differentiable real functions on $\mathbb{R}^n$, prove that

$$\nabla(fg) = f\nabla g + g\nabla f,$$

and that for $f \ne 0$,

$$\nabla(1/f) = -\frac{\nabla f}{f^2}.$$