

**MATH 4155/9063 FALL 2013**

HOMEWORK ASSIGNMENT 4. DUE NOVEMBER 22.

- 4.1. Suppose that a function  $\phi$  is real analytic on  $\mathbb{R}^n$  and is a test function. Prove that  $\phi \equiv 0$ .
- 4.2. Suppose that  $\phi \in \mathcal{D}(\Omega)$  for a domain  $\Omega \subset \mathbb{R}^n$ . Assume that  $\phi \geq 0$ , and that  $K = \text{supp } \phi$  is compact in  $D$ . Determine  $\text{supp } \phi_\epsilon$ , where  $\phi_\epsilon$  is the regularization of  $\phi$ .
- 4.3. Suppose that  $f$  and  $g$  are test functions on  $\mathbb{R}^n$ . Prove that  $f * g = g * f$ .
- 4.4. Suppose  $(f_k)$  is a sequence of continuous functions converging to a function  $f$  in  $\mathcal{C}(\mathbb{R})$ . Prove that  $(f_k)$  converges to  $f$  in  $\mathcal{D}'(\mathbb{R})$ .
- 4.5. (Math 9063 only) Prove that

$$\lim_{n \rightarrow \infty} \sin(nx) = 0$$

in  $\mathcal{D}'(\mathbb{R})$ , but

$$\lim_{n \rightarrow \infty} \sin^2(nx) \neq 0.$$

Conclude that multiplication of distributions is not a continuous operation even where it is defined.