MATH 4155/9063 FALL 2013

HOMEWORK ASSIGNMENT 5. DUE DECEMBER 6.

5.1. Prove that if a test function ϕ vanishes in a neighbourhood of the support of a distribution f, then $\langle f, \phi \rangle = 0$. Would it suffice if ϕ vanishes on the support of f?

5.2. Let f be a distribution with compact support and let P be a polynomial. Show that $P * f$ is a polynomial.

5.3. For $f, g \in \mathcal{D}'(\mathbb{R})$ prove that $f * g$ is a well-defined distribution if $\operatorname{supp} f \subset (a, \infty)$ and $\operatorname{supp} g \subset (b, \infty)$ for some $a, b \in \mathbb{R}$.

5.4. Prove that every distribution is the limit of a sequence of distributions with compact support.

5.5. (Math 9063 only) Prove that f given by

$$\langle f, \phi \rangle = \sum_{n=0}^{\infty} \phi^{(n)}(n), \quad \phi \in \mathcal{D}(\mathbb{R})$$

is a distribution of infinite order of singularity.