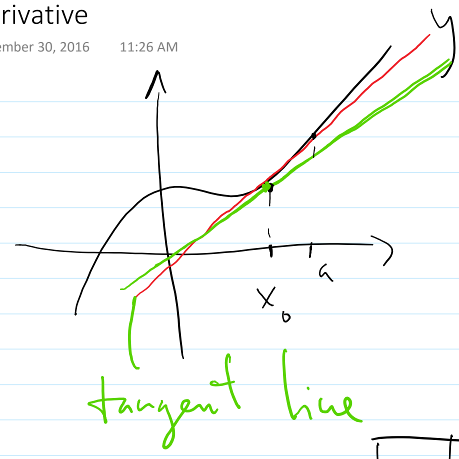
Derivative

September 30, 2016 11:26 AM



$$\frac{f(a)-f(x_0)}{a-x_0} = m$$

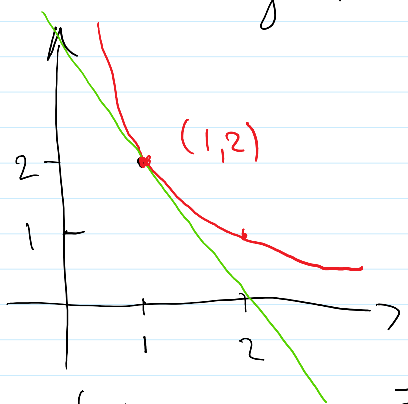
$$\text{slope of tangent line} = \lim_{a\to x_0} \frac{f(a)-f(x_0)}{\underbrace{a-x_0}_{h}} \qquad a = x_0+h$$

$$\boxed{f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}}$$

derivative of f at x_0 also/and slope of the tangent line

Example: $f(x)=y=\frac{2}{x}$ $\quad x_0=1$.

find $f'(1)$, and the equation of the tangent line passing through $(1,2)$.



$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2}{1+h}-2}{h} = \lim_{h\to 0} \frac{\frac{2-2(1+h)}{1+h}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2}-\cancel{2}-2\cancel{h}}{\cancel{h}(1+h)} = \lim_{h\to 0} \frac{-2}{1+h}$$

$$= \boxed{-2}$$

$$\underbrace{f'(1)}_{\text{So}} \quad \overbrace{f}$$

So $\left(\frac{2}{x}\right)'(1) = -2$.

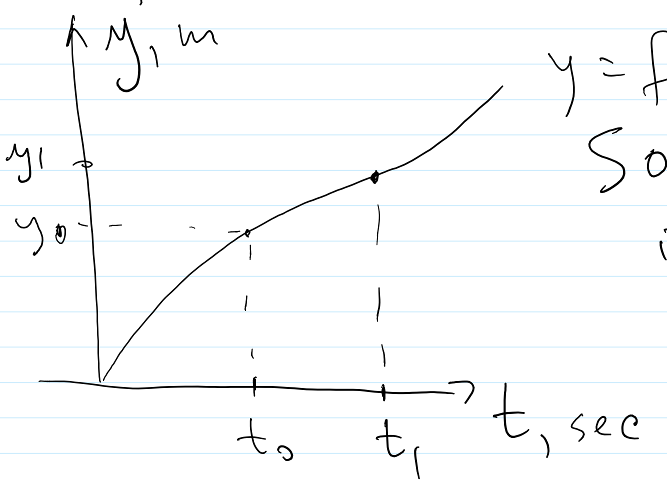
The tangent line passes through $(1,2)$ and has slope $= -2$. So its equation is

$$y-2 = -2(x-1)$$

$$\boxed{y = -2x+4}$$

$$\boxed{y = -2x + 4}$$

Suppose now that $y = f(t)$ represents the position function of an object at time t.



So, at time t_0 the object is distance y_0 from $\underbrace{\text{the reference points}}_{\substack{\shortparallel \\ \text{corresponds to } t=0}}$

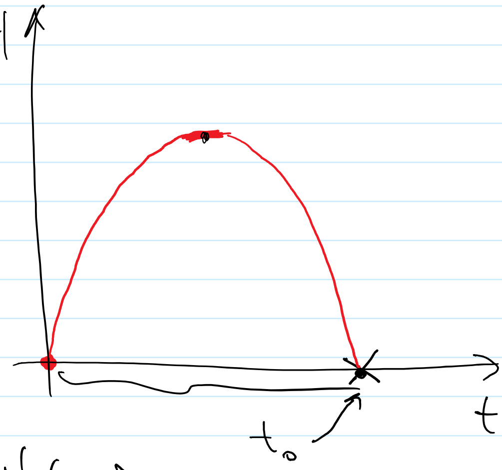
slope of f
velocity of the object over the interval $t_1 - t_0$.
$= \dfrac{y_1 - y_0}{t_1 - t_0}$ — distance / time

As $t_1 \to t$, slope of the tangent line ~ instantaneous velocity at time t_0.

$$\boxed{\text{So the velocity of the object} = \text{The derivative of the distance (or position function) of } f}$$

Velocity problem

September 30, 2016 11:26 AM



A rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height (in meters) after t seconds is given by H = 10t -1.86t^2.

1) Find the velocity of the rock after 1 second
2) Find the velocity at time t=a
3) When will the rock hit the surface
4) With what velocity will the rock hit the surface?

Solution.

(1) velocity at $t=1$ is $H'(1)$.

(2) velocity at $t=a$ is $H'(a)$.

(3) $H=0 \Rightarrow 10t-1.86t^2=0$

$t\cdot(10-1.86t)=0$

$\rightarrow \begin{cases} t=0, \text{ or} \\ \boxed{10-1.86t=0} \end{cases} \rightarrow \boxed{t_0=\frac{10}{1.86}\,(\text{sec})}$

(4) velocity $= H'(t_0)$

Ex: complete the solution by computing All the quantities.