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Technique of differentiation

1. $\frac{d}{dx}(c) = 0$

2. $\frac{d}{dx}(c\cdot f) = c\cdot\frac{d}{dx}f$

3. $\boxed{(x^n)' = n\cdot x^{n-1}}$, $n\in\mathbb{R}$

$n\in\mathbb{N}$, $(A+B)^n = \sum_{k=0}^{n}\binom{n}{k}A^{n-k}B^k = \underbrace{\binom{n}{0}A^n}_{k=0} + \underbrace{\binom{n}{1}A^{n-1}B}_{k=1}$

$+ \underbrace{\binom{n}{2}A^{n-2}B^2}_{k=2} + \dots + \underbrace{\binom{n}{n}B^n}_{k=n}$

$$\begin{array}{c} 1 \\ 1\ \ 1 \\ 1\ \ 2\ \ 1 \\ 1\ \ 3\ \ 3\ \ 1 \\ 1\ \ 4\ \ 6\ \ 4\ \ 1 \end{array}$$

$\boxed{\binom{n}{k} = \frac{n!}{(n-k)!\,k!}}$ binomial coefficients

$\boxed{n! = 1\cdot 2\cdot 3\cdots n}$

$\binom{4}{1} = \frac{4!}{3!\,1!} = 4$

$0! = 1$

Exercise:

$(x+y)^7 = \sum_{k=0}^{7}\binom{7}{k}x^{7-k}y^k$

$= \binom{7}{0}x^7 + \binom{7}{1}x^6y + \binom{7}{2}x^5y^2 + \dots$

$+ \binom{7}{7}y^7$

$\binom{7}{1} = \frac{7!}{6!\,1!}$

$\frac{7!}{5!\,2!} = \frac{6\cdot 7}{2} = 21$

$\frac{7!}{0!\,7!} = 1$

$$\boxed{= x^7 + 7x^6y + \dots}$$

- Exponential functions

$f(x) = b^x$ $\quad f'(x) = ?$ $\quad b>0$

$f'(x) = \lim_{h\to 0}\frac{b^{x+h}-b^x}{h} = \lim_{h\to 0}\frac{b^x\cdot(b^h-1)}{h}$

$$f'(x) = \lim_{h\to 0} \frac{b^{x+h} - b^{x}}{h} = \lim_{h\to 0} \frac{b^{x}\cdot(b^{h}-1)}{h}$$

$$= b^{x}\cdot \boxed{\lim_{h\to 0}\frac{b^{h}-1}{h}} = b^{x}\cdot f'(0)$$

$$f'(0) = \lim_{h\to 0}\frac{b^{h}-1}{h}$$

$f'(x)$ is proportional to $f(x)$.

Convenient definition:

$$\boxed{1 = \lim_{h\to 0}\frac{e^{h}-1}{h}}$$

This formula defines number e.

Now compute $(e^{x})'$

$$(e^{x})' = e^{x}\cdot 1 = e^{x}.$$

$$\boxed{(e^{x})' = e^{x}}$$

- $(f\pm g)' = f' \pm g'$.
- Product Rule: $(f\cdot g)' = f'\cdot g + g'\cdot f$

  Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'\cdot g - g'\cdot f}{g^{2}}$

Examples: $(\sqrt{x}\,e^{x})' = \underbrace{(\sqrt{x})'}_{x^{\frac{1}{2}}}e^{x} + \sqrt{x}\cdot(e^{x})'$

$$= \frac{1}{2}x^{-\frac{1}{2}}\cdot e^{x} + \sqrt{x}\,e^{x}$$

$$= \boxed{e^{x}\left(\frac{1}{2\sqrt{x}} + \sqrt{x}\right)}$$

$$= x^{-\frac{1}{2}}e^{x}\frac{1}{2}\cdot(1+2x)$$

$$(x^{2}-2)' \qquad 2x(x^{2}+1) - (x^{2}-2)\cdot 2x$$

$$\left(\frac{x^2-2}{x^2+1}\right)' = \frac{2x(x^2+1)-(x^2-2)\cdot 2x}{(x^2+1)^2}$$

$$= \frac{\cancel{2x^3}+2x-\cancel{2x^3}+4x}{(x^2+1)^2}$$

$$= \frac{6x}{(x^2+1)^2}$$

Derivative of $\sin x$? By definition

$$(\sin x)' = \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h} =$$

$$\lim_{h\to 0} \frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$$

$\sin x(\ldots)$

$$= \lim_{h\to 0}\left[\frac{\sin x\cos h-\sin x}{h} + \frac{\cos x\sin h}{h}\right]$$

$$\sin x \underbrace{\lim_{h\to 0}\left[\frac{\cos h-1}{h}\right]}_{\to 0} + \cos x \underbrace{\lim_{h\to 0}\frac{\sin h}{h}}_{\to 1}$$

$$\boxed{(\sin x)' = \cos x}$$