October 17, 2016 1:34 PM

Implicit differentiation

$e^y = \sin(y^2+x)$. Find $\frac{dy}{dx}$

Differentiate both sides:

$$e^y \cdot y' = \cos(y^2+x) \cdot \underbrace{(y^2+x)'}_{2y\cdot y'+1}$$

$$e^y \cdot y' = \cos(y^2+x)\cdot 2y \cdot y' + \cos(y^2+x)$$

solve for y':

$$y'(e^y - 2y\cos(y^2+x)) = \cos(y^2+x)$$

$$\boxed{y' = \frac{\cos(y^2+x)}{e^y - 2y\cos(y^2+x)}}$$

Derivatives of inverse trig functions

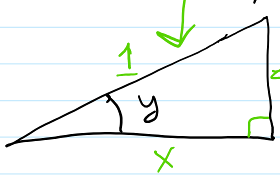
$y = \cos^{-1} x$ Find $y' = ?$

$\boxed{y = \cos^{-1} x} \iff x = \cos y$ Diff. implicitly:

$$1 = -\sin y \cdot y' \Rightarrow y' = -\frac{1}{\sin y}$$

$$\Rightarrow y' = -\frac{1}{\sin(\cos^{-1} x)} = -\frac{1}{\frac{\sqrt{1-x^2}}{1}} \Rightarrow$$

$\sin(\cos^{-1} x) = ?$ $\sqrt{1-x^2}$



$\cos y = \frac{x}{1}$

$\sin y = \frac{\sqrt{1-x^2}}{1}$

$$\boxed{y' = -\frac{1}{\sqrt{1-x^2}}}$$

$y = \cos^{-1} x$

x $\quad \sin y = \dfrac{\sqrt{1-x^2}}{1}$ $\quad y = \cos^{-1} x$

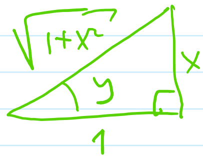
$y = \tan^{-1} x$ $\quad x = \tan y$ $\quad$ Use Implicit Differentiation

$\Rightarrow \quad 1 = (\tan y)' \Rightarrow 1 = \dfrac{1}{\cos^2 y} \cdot y'$

$\Rightarrow \quad y' = \cos^2 y \Rightarrow$

$y' = \left(\dfrac{1}{\sqrt{1+x^2}}\right)^2 \Rightarrow$

$$\boxed{(\tan^{-1} x)' = \frac{1}{1+x^2}}$$



$\tan y = \dfrac{x}{1}$

$\cos y = \dfrac{1}{\sqrt{1+x^2}}$

e.g.: $\quad \tan^{-1}(yx) = y \quad$ Find $\dfrac{dy}{dx}$.

Use Implicit Differentiation:

$$\frac{1}{1+(yx)^2} \cdot \underbrace{(yx)'}_{y' \cdot x + y \cdot 1} = y'$$

$$\frac{1}{1+y^2x^2} \cdot y'x + \frac{y}{1+y^2x^2} = y'$$

$$y'\left(\frac{x}{1+y^2x^2} - 1\right) = -\frac{y}{1+y^2x^2}$$

$$\boxed{y' = -\frac{y/(1+y^2x^2)}{\frac{x}{1+y^2x^2} - 1}}$$

3.6 Derivatives of logs

$$\frac{d(b^t)}{dt} = b^t \cdot \ln b$$

$y = \log_b x$, Find y'.

$$b^y = x \overset{\text{I.D.}}{\Longrightarrow} b^y \cdot \ln b \cdot y' = 1$$

$$y' = \frac{1}{b^y \ln b} \Rightarrow \boxed{(\log_b x)' = \frac{1}{x \ln b}}$$

if $b = e$, then $\boxed{(\ln x)' = \frac{1}{x}}$

<u>Example:</u> $y = x^x \qquad y' = ?$

chain rule

product rule

$$y = (e^{\ln x})^x = e^{x \ln x}$$

$$y' = (e^{x\ln x})' = e^{x \ln x}(x \cdot \ln x)' = e^{x \ln x}\left(\ln x + x \cdot \frac{1}{x}\right)$$

$$= e^{x \ln x}(\ln x + 1) = \underline{x^x(\ln x + 1)}$$

<u>Logarithmic differentiation</u>

$\log(a b) = \log a + \log b$

$\log a^n = n \cdot \log a$

<u>Example:</u> $y = \dfrac{x^{\frac{3}{2}}\sqrt{x^4+1}}{(3x^2+2)^3}$, $y' = ?$

<u>First take logarithm of both sides:</u>

$$\ln y = \ln\left(x^{\frac{3}{2}}\sqrt{x^4+1}\right) - \ln(3x^2+2)^3$$

$$= \frac{3}{2}\ln x + \frac{1}{2}\ln(x^4+1) - 3\ln(3x^2+2)$$

Use implicit differentiation:

$$\frac{1}{y} \cdot y' = \frac{3}{2}\frac{1}{x} + \frac{1/2}{x^4+1} \cdot (4x^3) - 3\frac{1}{3x^2+2} \cdot (6x)$$

$$y' = y\left[\frac{3}{2x} + \frac{2x^3}{x^4+1} - \frac{18x}{3x^2+2}\right]$$

$$y' = \frac{x^{\frac{3}{2}}\sqrt{x^4+1}}{(3x^2+2)^3} \cdot \left[\frac{3}{2x} + \frac{2x^3}{x^4+1} - \frac{18x}{3x^2+2}\right].$$