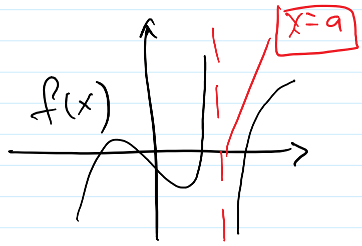
1. Things to recall:

(i) Trigonometric formulas: double-angle, $\frac{1}{2}$-angle, sums ($\sin(\alpha+\beta)$, ...).

(ii) properties of logs and exponents.

$\log(a\cdot b) =$ $(a^x)^y = \dots$, $e^{\ln a} = \dots$

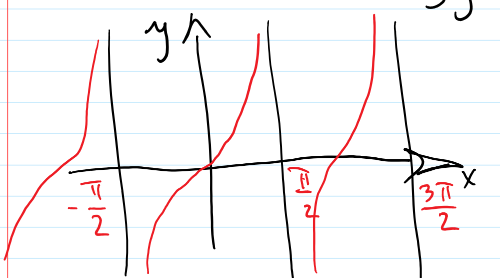
(iii) asymptotes



- $x=a$ is a vertical asymptote for $f(x)$ if $\lim\limits_{x\to a^{\pm}} f(x) = \pm\infty$

- $y=b$ is a horizontal asymptote if $\lim\limits_{x\to\pm\infty} g(x) = b$.

Example: $y = \tan(2\sin x)$. Find all vertical asymptotes of this function on $-\pi < x < \pi$.



if $k=-1$ $\frac{\pi}{2} - \pi = -\frac{\pi}{2}$

Solution: vertical asymptotes of $\tan x$ are $\boxed{x = \frac{\pi}{2} + \pi\cdot k}$, $k = 0, \pm 1, \pm 2, \dots$ (or $k \in \mathbb{Z}$)

- $2\sin x$ has no vertical asymptotes
- So $\tan(2\sin x)$ has vertical asymptotes at points where $2\sin x = \frac{\pi}{2} + \pi k$, $k \in \mathbb{Z}$.

or $\boxed{\sin x = \frac{\pi}{4} + \frac{\pi}{2}\cdot k.}$

$-1 \le \sin x \le 1$ so

$-1 \le \frac{\pi}{4} + \frac{\pi}{2}k \le 1$

$k=0 \to \frac{\pi}{4}$ ✓ $\quad \frac{\pi}{4} \dots$

$$-1 \le \frac{\pi}{4} + \frac{\pi}{2}k \le 1$$

$\pi \approx 3.14$

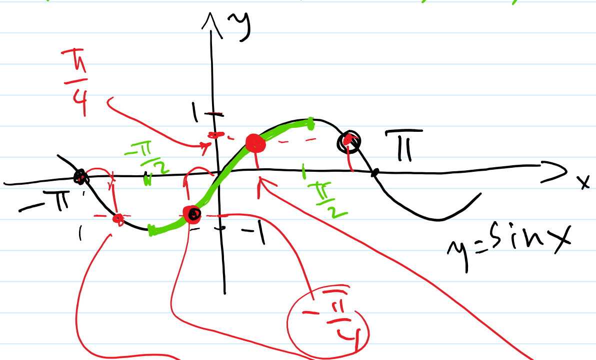
$k=0 \to \frac{\pi}{4}$ ✓

$k=1 \to \frac{\pi}{4}+\frac{\pi}{2} = \frac{3\pi}{4} > 1$

$k=-1 \to \frac{\pi}{4}-\frac{\pi}{2} = -\frac{\pi}{4}$ ✓

$k=-2 \to \frac{\pi}{4}-\pi = -\frac{3\pi}{4} < -1$

We solve: $\sin x = \frac{\pi}{4}$ or $\sin x = -\frac{\pi}{4}$



Red points are solutions.

$\sin x = \frac{\pi}{4}$

$\boxed{x = \sin^{-1}\frac{\pi}{4}}$

$\boxed{x = \pi - \sin^{-1}\frac{\pi}{4}}$

$\sin x = -\frac{\pi}{4}$

$\boxed{x = \sin^{-1}\left(-\frac{\pi}{4}\right) = -\sin^{-1}\frac{\pi}{4}}$

$\boxed{x = -\pi + \sin^{-1}\left(\frac{\pi}{4}\right)}$

Domain and range of $\sin^{-1}x$:

range of $\sin^{-1}$: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

domain $= [-1, 1]$

$$\tan(2\sin x) = \tan(\ldots)$$

(iv) Rules of differentiation

Chain Rule: $\boxed{[f(g(x))]' = f'(g(x))\, g'(x).}$

Example: Suppose $h(x) = f(g(x))$

find $\boxed{h'(1)}$ if

- $f(1) = 2$ • $g'(1) = 7$

- $f'(1) = 3$
- $f(2) = 9$
- $f'(2) = -2$
- $g(1) = 2$

Solution: $h'(1) = f'(g(1)) \cdot g'(1)$
$$= f'(2) \cdot 7 = -2 \cdot 7 = \underline{\underline{-14}}$$

(c) Undetermined expressions:

[be careful when compute limits)

$\boxed{\frac{0}{0}}$, $\lim\limits_{x \to 0} \frac{x^2}{x} = 0$ $\quad \lim\limits_{x \to 0^+} \frac{x}{x^2} = \infty$

$\lim\limits_{x \to 0} \frac{x^2}{3x^2} = \frac{1}{3}$; $\boxed{\lim\limits_{x \to 0} \frac{\sin x}{x} = 1}$

$\boxed{\frac{\infty}{\infty}}$ Similar. take the above limits as $x \to \infty$.

$\boxed{0 \cdot \infty}$ $\lim\limits_{x \to 0} \underbrace{\frac{1}{x}}_{\infty} \cdot \underbrace{\sin x}_{\infty} = 1.$

undefined $\to \boxed{1^\infty}$ $\lim\limits_{n \to \infty} \underbrace{\left(1 + \frac{1}{n}\right)}_{1}^{n \to \infty} = e$

$(x^a)^a = x^{a^2}$

$\lim\limits_{n \to \infty} \left(1 + \frac{1}{n}\right)^{n^2} = \lim\limits_{n \to \infty} e^n = \infty.$

$n \cdot n$
$\parallel$
$\left(1 + \frac{1}{n}\right)^{n^2} = \left(1 + \frac{1}{n}\right)^{n \cdot n} = \left(\left(1 + \frac{1}{n}\right)^n\right)^n$

$$\left(1+\frac{1}{n}\right) = \left(1+\frac{1}{n}\right)^{n\cdot n} = \left[\left(1+\frac{1}{n}\right)^{n}\right].$$