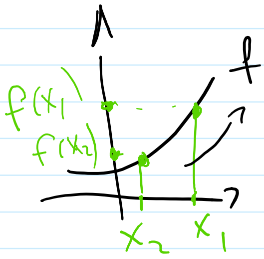
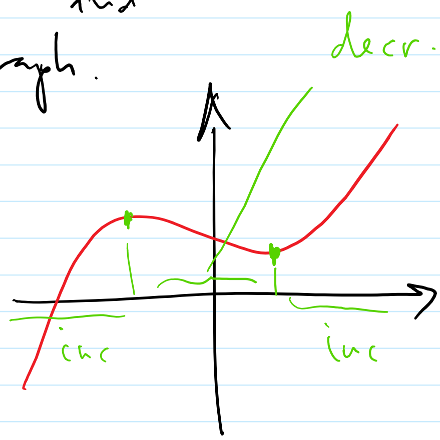
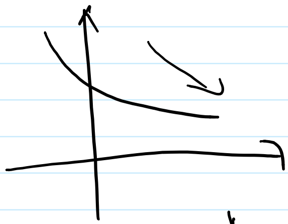
October 31, 2016 1:28 PM

4.3 Derivative of a function and the shape of its graph.



f is increasing





f is decreasing

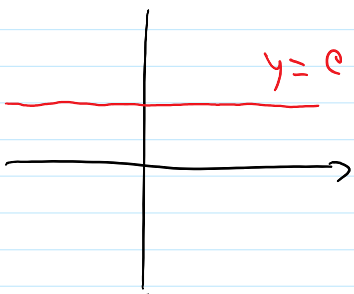
Def: A function $f(x)$ is increasing on $[a,b]$ if $f(x_1) \geq f(x_2)$ if $x_1 > x_2$, $x_{1,2} \in [a,b]$

Def $f(x)$ is strictly increasing if $f(x_1) > f(x_2)$ for $x_1 > x_2$

Similarly we define a decreasing (strictly decreasing function).

is strictly increasing on $[0, \infty)$

is strictly decreasing on $(-\infty, 0]$

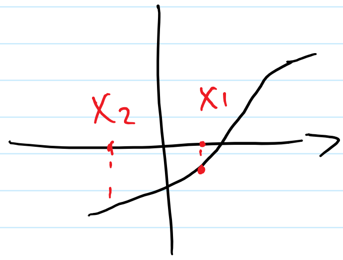


increasing and decreasing (not strictly).

Test:
- If $f'(x) > 0$ then $f(x)$ is increasing.
- If $f'(x) < 0$ then $f(x)$ is decreasing.

Proof: Say $f'(x) > 0$. Let $x_1 > x_2$. Apply the

Proof: Say $f'(x) > 0$. Let $x_1 > x_2$. Apply the Mean Value Theorem:



There is $c \in (x_2, x_1)$ such that

$$\boxed{\underbrace{f'(c)}_{>0} = \frac{f(x_1) - f(x_2)}{\underbrace{x_1 - x_2}_{>0}}} \Rightarrow$$

$$f(x_1) - f(x_2) > 0 \Rightarrow$$
$$f(x_1) > f(x_2). \qquad \square$$

Example 1: $\boxed{f(x) = 3x^4 - 4x^3 - 12x^2 + 5.}$

Find the intervals where $f(x)$ is incr. and where decreasing.

Solution: $f'(x) = 12x^3 - 12x^2 - 24x$

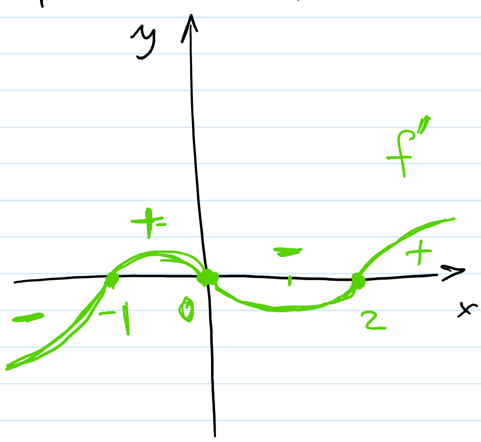
To find where $f' > 0$ and < 0, find $f' = 0$.

$\rightarrow$ $12x^3 - 12x^2 - 24x = 0$

$12x(x^2 - x - 2) = 0$

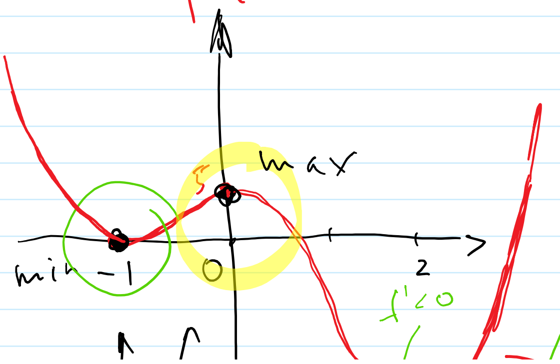
$x = 0$ $\qquad (x+1)(x-2)$

$x = -1$ $\qquad x = 2$



$\Rightarrow$ $f(x)$ is increasing on $[-1, 0] \cup [2, \infty)$ $\quad (f' > 0)$

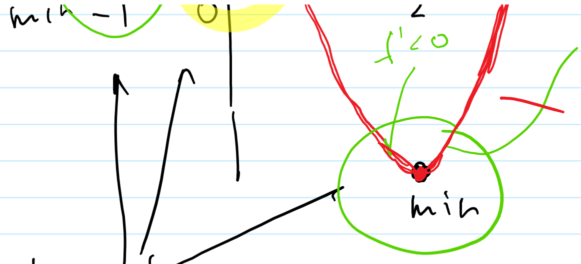
$f(x)$ is decreasing on $(-\infty, -1] \cup [0, 2]$ $\quad (f' < 0)$



$f(0) = 5$

$f(-1) = 3 + 4 - 12 + 5 = 0$

$f(2) = 48 - 32 - 48 + 5 = -27$

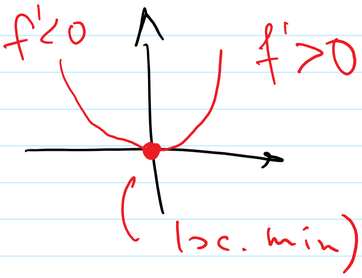


rough sketch of the graph of $f(x)$.

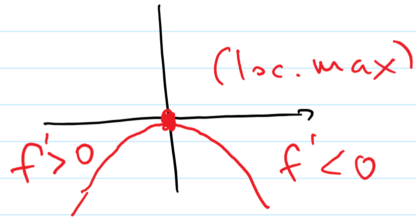
local max and min.

- If $f'(x)$ changes the sign at a point a from "$-$" to "$+$" $\left(\begin{array}{l} f'(x)<0 \text{ for } x<a \\ f'(x)>0 \text{ for } x>a \end{array}\right)$ then a is a local minimum.
- If $f'(x)$ changes the sign at a from $+$ to $-$, then a is a loc. max for $f(x)$.

<u>Model example:</u> $\to y = x^2$
$y' = 2x$



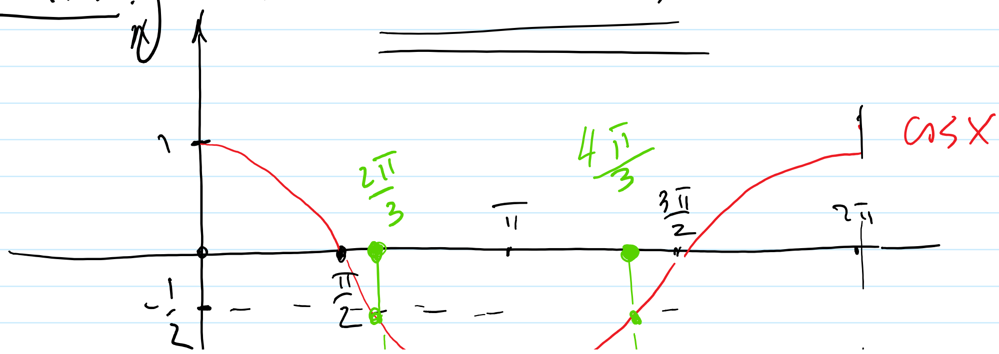
And $y = -x^2$
$y' = -2x$

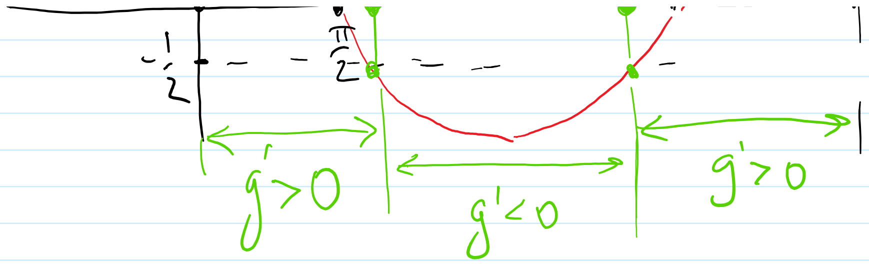


<u>Example 2.</u> $g(x) = x + 2\sin x$, $x \in [0, 2\pi]$.

Find where g is incr/decr, loc. max/min.

<u>Solution:</u> $g'(x) = \underline{\underline{1 + 2\cos x = 0}} \Rightarrow \cos x = -\frac{1}{2}$.





So on the interval $[0, 2\pi]$ the function

$x + 2\sin x$ is

- increasing on $[0, \frac{2\pi}{3}] \cup [\frac{4\pi}{3}, 2\pi]$
- decreasing on $[\frac{2\pi}{3}, \frac{4\pi}{3}]$.