November 9, 2016 1:19 PM

Problem 3: Find an equation of the line through the point (3,5) that cuts off the least area from the first quadrant

[Continued] $A(m) = -\frac{1}{2}\frac{(3m-5)^2}{m}$, $m<0$ $= 9m^2 - 30m + 25$

$$A'(m) = -\frac{1}{2}\frac{(18m-30)m - (9m^2-30m+25)}{m^2} = 0$$

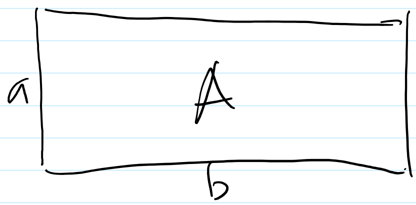
$$\Rightarrow 18m^2 - \cancel{30m} - 9m^2 + \cancel{30m} - 25 = 0$$

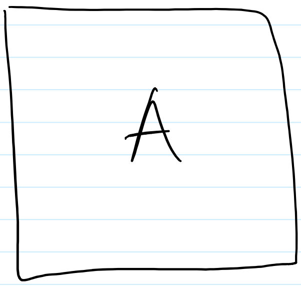
$$\Rightarrow 9m^2 - 25 = 0 \qquad m = \pm\frac{5}{3}$$

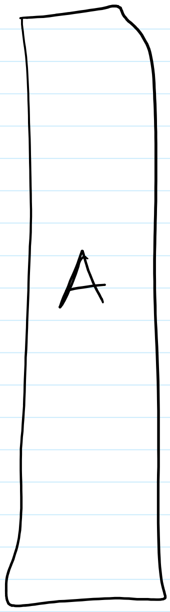
$$\boxed{m = -\frac{5}{3}} \Rightarrow y = mx + (5 - 3\cdot m)$$

$$\Rightarrow \boxed{y = -\frac{5}{3}\cdot x + 10}$$

Problem 4: Show that of all the rectangles with a given area, the one with smallest perimeter is a square.







$A = a\cdot b$ - constraint $\Rightarrow b = \frac{A}{a}$

$P = 2(a+b)$ - is to minimize

$$P(a) = P = 2\left(a + \frac{A}{a}\right) = \frac{2(a^2+A)}{a}$$

To find the global minimum,

$$\frac{dP}{da} = 2\cdot\frac{2a^2 - (a^2+A)}{a^2} = 0 \qquad \frac{dP}{da} = 2\frac{a^2-A}{a^2}$$

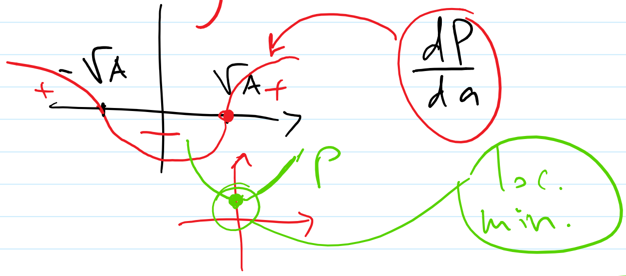
$$a^2 - A = 0 \Rightarrow a = \pm\sqrt{A}.$$

So $a = \sqrt{A}$, $b = \frac{A}{a} = \frac{A}{\sqrt{A}} = \sqrt{A}$.

Since $a = b$, the smallest perimeter is of a square

Since $a=b$, the smallest perimeter is of a square.

Why is this a min, not max?



4.9. Antiderivatives

Def: A function $F(x)$ is called an antiderivative of $f(x)$ on the interval $I=[a,b]$, if

$$F'(x) = f(x) \text{ for all } x \in I.$$

Examples: 1) $f(x) = x^2$

An antiderivative of $f(x)$ is

$$F(x) = \frac{1}{3}x^3 \qquad F'(x) = \left(\frac{1}{3}x^3\right) = x^2 = f(x)$$

$$F_1(x) = \frac{1}{3}x^3 + 13$$

2) $f(x) = x^4$

$$F(x) = \frac{1}{5}x^5, \quad F'(x) = \frac{1}{\cancel{5}}\cdot\cancel{5}\cdot x^4 = x^4.$$

$F_1(x) = \frac{1}{5}x^5 + C$, C = some constant.

So Antiderivative is not a unique function.

Theorem: if $F_1(x)$ and $F_2(x)$ are antiderivatives of $f(x)$, then there exists a constant C s.t. $F_1(x) = F_2(x) + C$.

Proof: Consider $F(x) = F_1(x) - F_2(x)$. Then

$$F'(x) = F_1'(x) - F_2'(x) = f(x) - f(x) = 0.$$

Claim: if $F'(x) = 0$ for all x, then

$F = \text{const}.$

This follows from the Mean Value Theorem.
Indeed, if $F(x_1) \neq F(x_2)$ for some x_1, x_2
Then by the MVT: $\boxed{F(x_2) - F(x_1) = \underset{0}{\underset{\|}{F'(c)}}(x_2 - x_1)}$
for some $c \in (x_1, x_2)$.

$\Rightarrow F(x_1) = F(x_2)$. $\Rightarrow F(x) = \text{const}$. $\square$

So if $F(x)$ is some antiderivative of $f(x)$
Then $F(x) + C$ is the most general form of antiderivative of f.

Function $f(x)$	Antiderivative of $f(x)$
x^n	$\frac{1}{n+1} x^{n+1} + C, \quad n \neq -1.$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
e^x	$e^x + C$

$(\ln x)' = \frac{1}{x}$ true for $x > 0$

$\boxed{(\ln|x|)' = \frac{1}{x} \text{ for } x \neq 0}$

$\ln|x| = \begin{cases} \ln x, & x > 0 \\ \ln(-x), & x < 0 \end{cases}$

$(\ln|x|)' = \begin{cases} \frac{1}{x}, & x > 0 \\ \frac{1}{-x} \cdot (-1) = \frac{1}{x}, & x < 0 \end{cases}$

$\cancel{\ln(-x) = \ln(-1 \cdot x) = \ln(-1) + \ln x}$

$\frac{1}{1+x^2}$	$\tan^{-1} x + C$

$(e^{x+c})' = e^{x+c} \neq e^x$ if $c \neq 0$