Antiderivatives

$F(x)$ is antiderivative of $f(x)$, if

$F'(x) = f(x)$.

- F is not unique, in fact, if $F_1(x)$ is a particular antiderivative, then any antiderivative has the form $\boxed{F(x) = F_1(x) + C}$, $c = \text{const}$.

most general form of antiderivative

Examples: (1) Antider. of $\sin x$ is $\underline{-\cos x + C}$

(2) Antider've of $e^x + x^2$ is

$$e^x + \frac{1}{3}x^3 + C.$$

etc.

Problem 1: If $f'(x) = e^x + 2016x$ find $f(x)$ s.t. $f(0) = 2017$.

Solution: Need to find antiderivative of $f'(x)$.

$$e^x + 2016x \longrightarrow f(x) = \underbrace{e^x + \frac{2016}{2}x^2 + C}_{\text{family of functions}}$$

$$f(0) = e^0 + \frac{2016}{2} \cdot 0^2 + C = \boxed{1 + C = 2017}$$

equation for C.

$\Rightarrow C = 2016$.

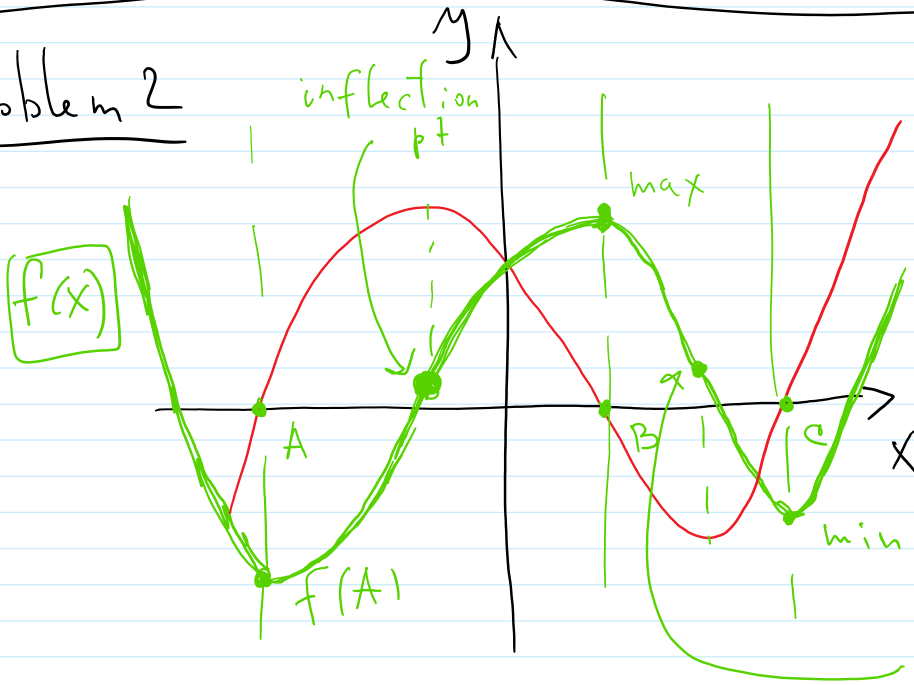
Answer: $\boxed{e^x + 1008x^2 + 2016} = f(x)$.

check: $f(0) = e^0 + 2016 = 2017$ ✓

check: $f(0) = e^0 + 2016 = 2017$ ✓

$f'(x) = e^x + 2016x$ ✓

Problem 2



$y = f'(x)$

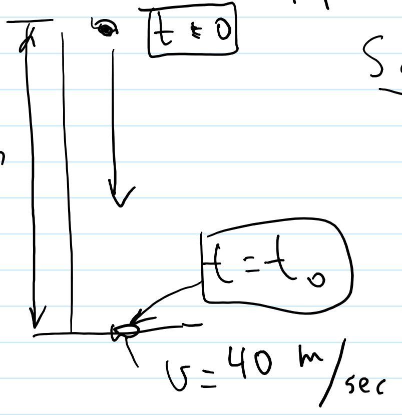
Sketch the graph of f(x).

The graph of f(x) can be moved up or down to obtain other antiderivatives of $f'(x)$.

Problem 3: A rock falls from a cliff.

When it hits the ground its velocity is 40 m/sec. What is the height of the cliff?

t = time



Solution: $s(t)$ = distance traveled by the rock.

$s(0) = 0.$

$s'(t)$ = velocity

$s'(0) = 0$

$s'(t_0) = 40$, $t_0 = ?$

$s''(t) = g$ $\quad(\approx 10\ m/sec^2)$ const.

$s''(t) = 10$

$\boxed{s'(t) = 10t}$ we can find t_0 from here

$s'(t_0) = 10 \cdot t_0 = 40$ $\boxed{t_0 = 4 \text{ sec}}$

Need to find $s(4)$.

$s(t) = 5t^2 + C$, $s(0) = 0 \Rightarrow C = 0$

$\boxed{\text{So } s(t) = 5t^2, \quad h = s(4) = 5 \cdot 4^2 = 80.}$