Sigma notation

Sigma

- $1+2+3+4+5+\dots+99+100 = \sum_{i=1}^{100} i$
- $1^2+2^2+3^2+4^2+\dots+100^2 = \sum_{i=1}^{100} i^2$
- $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\dots+\frac{99}{100} = \sum_{i=1}^{99} \frac{i}{i+1}$
- $\sum_{i=1}^{10}\left[\underbrace{(i+1)^2-i^2}\right] = (2^2-1^2)+(3^2-2^2)+\dots+(11^2-10^2)$

$$= \cancel{2^2}-1^2+\cancel{3^2}-\cancel{2^2}+\cancel{4^2}-\cancel{3^2}+\dots+\cancel{10^2}-\cancel{9^2}+11^2-\cancel{10^2}$$

$$= 11^2-1^2 = 120$$

"Telescoping sum"

- $\boxed{1+2+\dots+(n-1)+n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$

e.g. $1+2+\dots+10 = \frac{10(11)}{2} = 55$.

Proof:

$$\begin{array}{rl} & 1+2+3+4+\dots+(n-2)+(n-1)+n \\ + & n+(n-1)+(n-2)+\dots+3+2+1 \\ \hline & (n+1)+(n+1)+(n+1)\ \dots\ (n+1)+(n+1)+(n+1) \end{array}$$

$$=(n+1)\cdot n$$

$$\Rightarrow 1+2+\dots+n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad \square$$

Recall: $(a+b)^n = a^n + \binom{1}{n}a^{n-1}b + \binom{2}{n}a^{n-2}b^2+\dots+b^n$

$$= \sum_{k=0}^{n} \binom{k}{n} a^{n-k} b^k.$$

Leibnitz formula: $(f\cdot g)' = f'\cdot g + f\cdot g'$,

$$(f\cdot g)'' = (f'\cdot g + f\cdot g')' =$$

$$f''\cdot g + f'\cdot g' + f'\cdot g' + f\cdot g''$$

$$f''g + f'g' + f'g' + f\cdot g'' = f''g + 2f'\cdot g' + f\cdot g''$$

$$(f\cdot g)''' = \dots$$

$$\boxed{(f\cdot g)^{(n)} = \sum_{k=0}^{n} \binom{k}{n} f^{(n)}\cdot g^{(n-k)}}$$

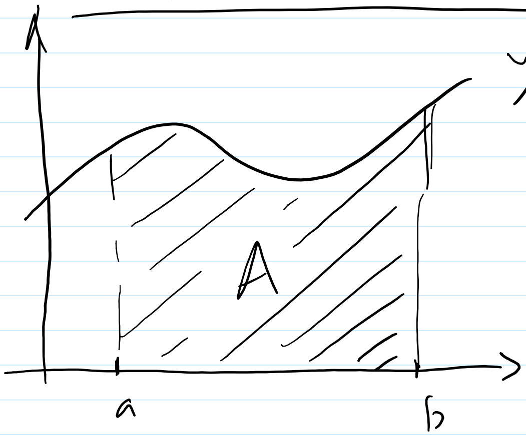
$g^{(0)} = g$

e.g. $(f\cdot g)^{(4)} = f^{(4)}\cdot g + 4f^{(3)}\cdot g' + 6f''\cdot g'' + 4f'g^{(3)} + f\cdot g^{(4)}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

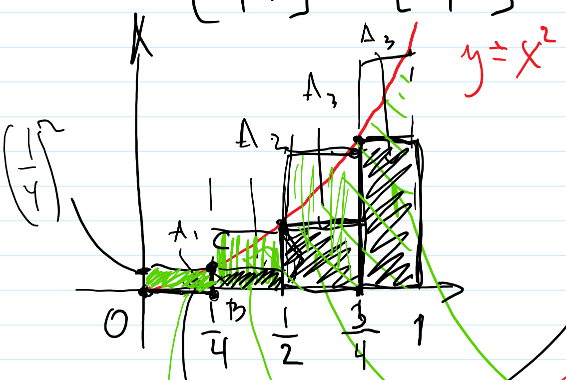
Appendix E

5.1 Area and Distances



A = area under the graph = ?

To be more specific, take $f(x) = x^2$
$[a,b] = [0,1]$



$A_1 \simeq$ area of a ~~ΔABC.~~
$A_2 \simeq$ area of a
A_3, A_4 — same

smaller rectangle $< A_j <$ bigger rectangle

left end point on the ... use right-end point on the ...

left end point on the subinterval

point on the subinterval

$$\frac{1}{4}\cdot 0+\frac{1}{4}\cdot \underbrace{|BC|}_{}^{\left(\frac{1}{4}\right)^2}+\frac{1}{4}\cdot\left(\frac{1}{2}\right)^2+\frac{1}{4}\cdot\left(\frac{3}{4}\right)^2 = L_4$$ left-end sum

$$\frac{1}{4}\cdot\left(\frac{1}{4}\right)^2+\frac{1}{4}\cdot\left(\frac{1}{2}\right)^2+\frac{1}{4}\cdot\left(\frac{3}{4}\right)^2+\frac{1}{4}\cdot 1^2 = R_4$$ right-end sum

area under $f(x)=x^2 \approx L_4$ or R_4.

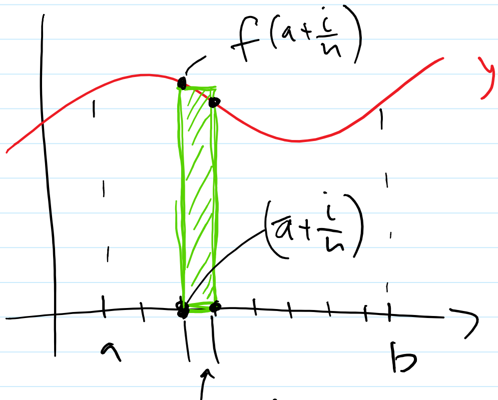
To be more precise we can divide the interval into 8 subintervals:

$$L_8 = \frac{1}{8}\cdot 0+\frac{1}{8}\cdot\left(\frac{1}{8}\right)^2+\frac{1}{8}\left(\frac{2}{8}\right)^2+\dots+\frac{1}{8}\cdot\left(\frac{7}{8}\right)^2$$

$$R_8 = \frac{1}{8}\cdot\left(\frac{1}{8}\right)^2+\frac{1}{8}\left(\frac{2}{8}\right)^2+\dots+\frac{1}{8}\cdot(1)^2$$

etc.

Def: The area under the graph of $f(x)$ on the interval $[a,b]$ is



$\Delta x = \frac{b-a}{n}$

$y=f(x)$

$$\boxed{\lim_{n\to\infty}\underbrace{\sum_{i=0}^{n-1} f(a+i\Delta x)\cdot\Delta x}_{=L_n}}$$

area.

$L_n = f(a)\cdot\Delta x + f(a+\Delta x)\Delta x + \dots + f(a+(n-1)\Delta x)\Delta x$

Alternatively, we may have

$$\text{Area} = \lim_{n\to\infty}\underbrace{\sum_{i=1}^{n} f(a+i\Delta x)\,\Delta x}_{R_n}$$

$a+n\Delta x = a+n\cdot\frac{b-a}{n} = b$

$$R_n = f(a+\Delta x)\cdot\Delta x + f(a+2\Delta x)\cdot\Delta x + \dots + f(b)\Delta x$$