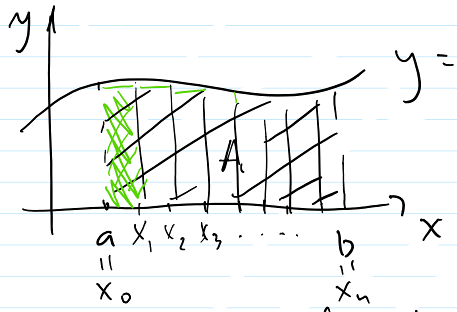
November 16, 2016 1:32 PM

Area under the graph



Divide $[a,b]$ into n subintervals of length $\Delta x = \frac{b-a}{n}$.

We have intervals: $[x_0,x_1], [x_1,x_2], \dots [x_{n-1},x_n]$

$$A \approx f(x_1)\cdot\Delta x + f(x_2)\cdot\Delta x + \dots + f(x_n)\cdot\Delta x = R_n$$

≥ 0 ≥ 0 ≥ 0

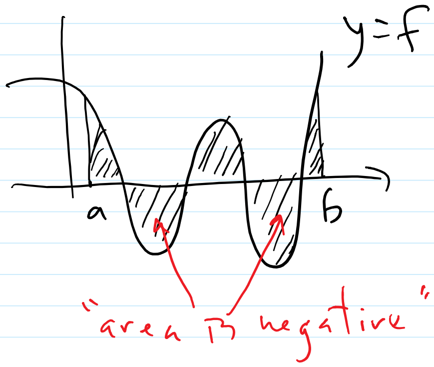
$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} R_n$$

or we could use left sums:

$$L_n = f(x_0)\Delta x + \dots + f(x_{n-1})\Delta x$$

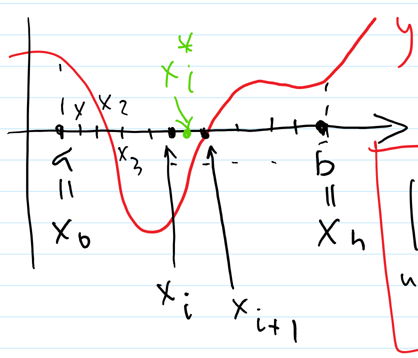
$$A = \lim_{n\to\infty} \sum_{i=0}^{n-1} f(x_i)\Delta x = \lim L_n$$

could be < 0.



Definite Integral

$|x_{i+1} - x_i| = \Delta x$



$x_i^* \in [x_i, x_{i+1}]$ sample point

$$\boxed{\lim_{n\to\infty} \sum_{i=0}^{n-1} f(x_i^*)\Delta x \overset{\text{def}}{=} \int_a^b f(x)\,dx}$$

Riemann sum

integrand

limits of integration

Riemann sum

integrand

limits of integration
a = lower limit
b = upper limit

$\int$ = integral sign

So the definite integral is the limit of Riemann sums

Def: We say that $f(x)$ is integrable on the interval $[a,b]$ if $\int_a^b f(x)\,dx$ exists,

i.e., the limit of Riemann sums exists.

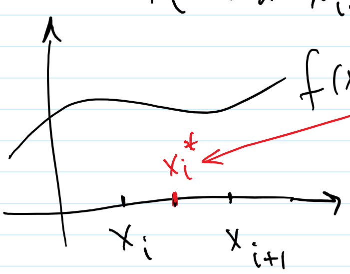
Theorem: If $f(x)$ is continuous on $[a,b]$, or has a finite number of discontinuities, then $f(x)$ is integrable on $[a,b]$.

f is bounded and

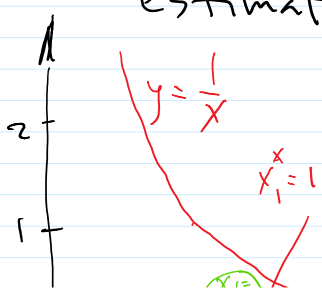
So, ~~all~~ for example, polynomials, e^x, $\ln x$, $\sin x$, are integrable where defined.

The Midpoint Rule: $\int_a^b f(x)\,dx$ can be approximated by choosing x_i^* to be the middle point between x_i and x_{i+1}.

$$\boxed{x_i^* = \frac{x_i + x_{i+1}}{2}}$$

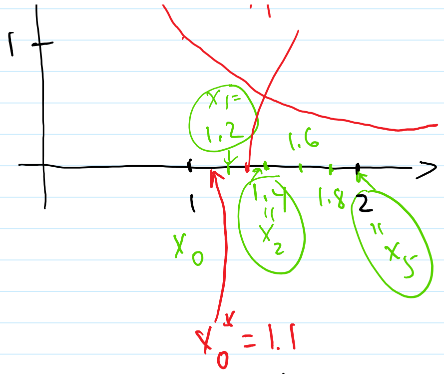


Example: Use the Midpoint Rule to estimate $\int_1^2 \frac{1}{x}\,dx$ with 5 subintervals.



$$\int_1^2 \frac{dx}{x} \simeq f(\overset{1.1}{\overset{\shortparallel}{x_0^*}})\Delta x + f(\overset{1.3}{\overset{\shortparallel}{x_1^*}})\Delta x + \dots$$

$$\dots f(x_4^*)\Delta x \quad \circleddash$$

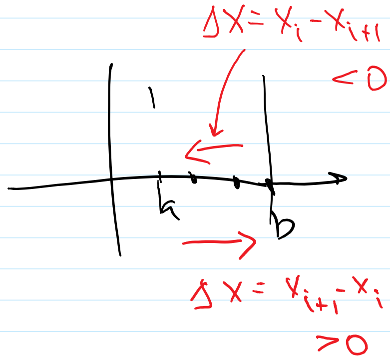


$\therefore f(x_4^*)\Delta x \;=\;$ ($x_4^* = 1.9$)

$$\Delta x = \frac{2-1}{5} = 0.2$$

$$= \frac{1}{5}\left[\frac{1}{1.1} + \frac{1}{1.3} + \frac{1}{1.5} + \frac{1}{1.7} + \frac{1}{1.9}\right].$$

Properties of Definite Integrals



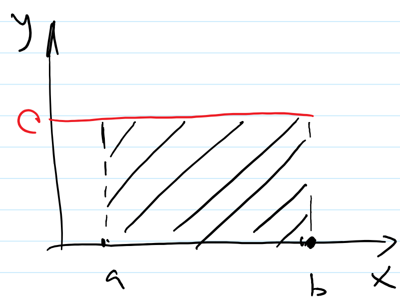
(a) $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

(b) $$\int_a^a f(x)\,dx = 0$$

(c) $$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

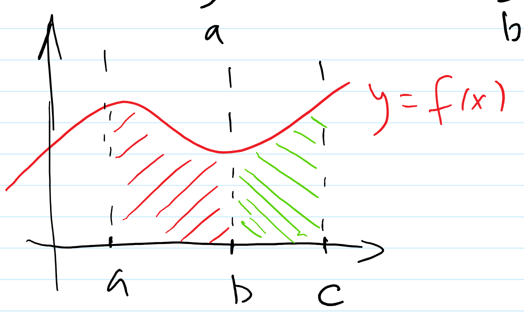
(true because $\sum_{i=1}^{n} (f+g)(x_i^*)\Delta x = \sum f(x_i^*)\Delta x + \sum g(x_i^*)\Delta x$)

(d) $$\int_a^b c\cdot f(x)\,dx = c\cdot \int_a^b f(x)\,dx$$



(e) $$\int_a^b c\,dx = c\cdot(b-a)$$

(f) $$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx, \quad a<b<c.$$



Example: Suppose $f(x)$ is an odd function i.e. $f(-x) = -f(x)$. Then function, i.e., " $f(-x) = -f(x)$. Then

$$\int_{-1}^{1} f(x)\,dx \overset{(f)}{=} \underbrace{\int_{-1}^{0} f(x)\,dx}_{x<0} + \int_{0}^{1} f(x)\,dx =$$

So $f(x) = -f(-x)$

$$\int_{-1}^{0} f(x)\,dx = -\int_{0}^{1} f(x)\,dx$$

$$= -\int_{0}^{1} f(x)\,dx + \int_{0}^{1} f(x)\,dx = 0$$