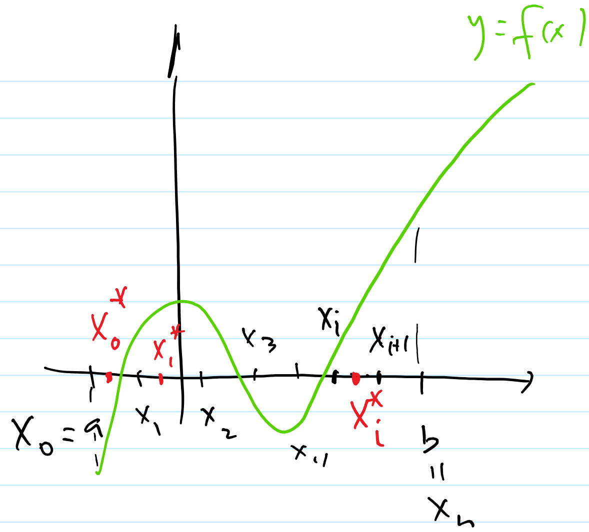
Definite Integral



Def:

$$\lim_{n\to\infty} \sum_{i=0}^{n-1} f(x_i^*)\,\Delta x = \underbrace{\int_a^b f(x)\,dx}$$

Riemann Sum

The definite integral of $f(x)$ over $[a,b]$.

$$\Delta x = \frac{b-a}{n}$$

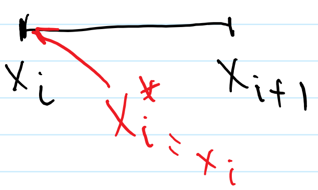
Example: $f(x) = x^2 + \frac{1}{x}$, $[a,b] = [2,5]$

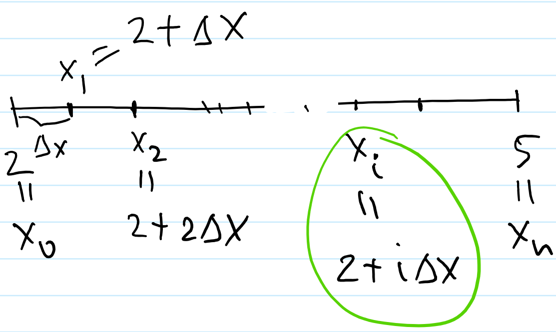
Express $\int_2^5 \left(x^2 + \frac{1}{x}\right)dx$ as a limit of Riemann sums.

$n \in \mathbb{N}$ $\quad \Delta x = \frac{5-2}{n} = \frac{3}{n}$ $\quad f(x_i^*) = (x_i^*)^2 + \frac{1}{x_i^*}$. So

$$\int_2^5 \left(x^2 + \frac{1}{x}\right)dx = \lim_{n\to\infty} \sum_{i=0}^{n-1} \left[\left((x_i^*)^2 + \frac{1}{x_i^*}\right)\frac{3}{n}\right].$$

We may use the left endpoint as x_i^* for each subinterval





If we use left-end points, then

$$\int_2^5 \left(x^2 + \frac{1}{x}\right)dx = \lim_{n\to\infty} \sum_{i=0}^{n-1} \left[(2+i\Delta x)^2 + \frac{1}{(2+i\Delta x)}\right]\frac{3}{n}$$

$$\Delta x = \frac{3}{n}$$

May use right endpoints ...

Properties (continued)

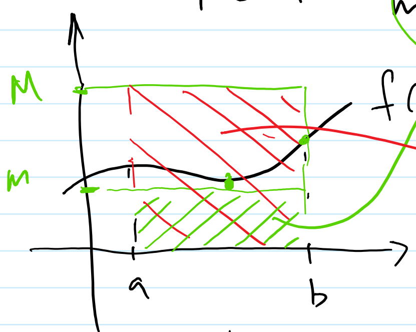
(g) if $f \geq 0$, then $\int_a^b f(x)\,dx \geq 0$, $a < b$.

More generally, if $f(x) \leq g(x)$, then

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx \quad , \quad a < b.$$

(h) If $f(x)$ is such that $m \leq f(x) \leq M$ for all $x \in [a, b]$ where $m, M \in \mathbb{R}$, $m < M$,

Then $$m(b-a) \leq \int_a^b f(x)\,dx \leq M \cdot (b-a)$$



$1 = e^0 < e^b$

$\frac{1}{x} > 0$

Example: Estimate $\int_1^5 e^{\frac{1}{x}}\,dx$

Sol'n: Use property (h). $\cancel{1}\ e^{\frac{1}{5}} \leq e^{\frac{1}{x}} \leq e^1$

Why? $1 < x < 5$

$$\underbrace{e^{\frac{1}{5}}(5-1)}_{4e^{\frac{1}{5}}} \leq \int_1^5 e^{\frac{1}{x}}\,dx \leq \underbrace{e\cdot(5-1)}_{4e}$$