November 23, 2016 1:33 PM

Fundamental Theorem of Calculus, II.

Thm 1: if $f(x)$ is continuous on $[a,b]$, then $F(x) = \int_a^x f(t)\,dt$ is continuous on $[a,b]$, differentiable on (a,b), and
$$F'(x) = f(x)$$

Thm 2: Let $f(x)$ be continuous on $[a,b]$. Then $\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$ for any $F(x)$ such that $F'(x) = f(x)$.

Proof: Suppose $g(x)$ is some antiderivative of $f(x)$. Then $F(x) = g(x) + c$. $g = F - c$

Define $g(x) := \int_a^x f(t)\,dt$. By Thm 1 $g(x)$ is an antiderivative of f.

$$g(b) = \int_a^b f(x)\,dx \qquad g(a) = \int_a^a f(x)\,dx = 0$$

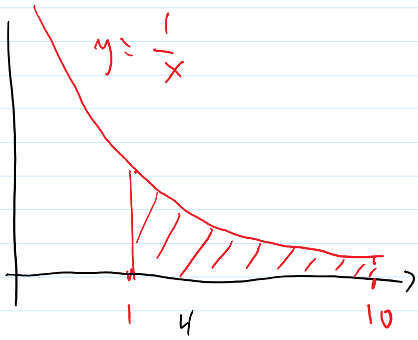
$$\int_a^b f(x)\,dx = g(b) - g(a) = (F(b) - \cancel{c}) - (F(a) - \cancel{c})$$
$$= F(b) - F(a) \qquad \square$$

Examples ① $\int_1^3 e^{2x}\,dx \overset{FTC}{=} F(3) - F(1)$, where $F'(x) = e^{2x}$, say, $F(x) = \frac{1}{2}e^{2x}$.

$$= \frac{1}{2}e^{2x}\Big|_1^3 = \frac{1}{2}e^6 - \frac{1}{2}e^2 = \boxed{\frac{e^6 - e^2}{2}}.$$

$$\left(\frac{1}{2}e^{2x}\right)' = \frac{1}{2}e^{2x}\cdot 2 = e^{2x}$$

(2) Find the area under the graph of $y = \frac{1}{x}$ from $x=1$ to $X=10$.



$$\text{Area} = \int_1^{10} \frac{1}{x}\,dx \overset{FTC}{=} \ln|x| \Big|_1^{10} = \ln 10.$$

antiderivative of $\frac{1}{x}$ is $\ln|x|$

(3) $\int_2^4 \ln x\,dx = ?$ Antiderivative of $\ln x$ is

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

Even for simple functions it is not easy to find an antiderivative...

(4) $\int_{-1}^{1} \frac{dx}{x^2} \overset{FTC}{=} \cancel{-\frac{1}{x}\Big|_{-1}^{1} = -1-(-1)=0}.$

In the FTC, in the expression $\int_a^b f(x)\,dx$ $f(x)$ must be continuous on $[a,b]$.
But $\frac{1}{x^2}$ is not defined at 0, in particular, $f(x)$ is not continuous on $[-1,1]$.
We cannot solve this problem.