November 25, 2016 1:26 PM

(5.4 Indef. Integral)

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if $f(x)$ is an integrable function, denote by $\int f(x)\,dx$ the family of antiderivatives of f and call it indefinite integral.

$\int f(x)\,dx$ = a function (a family of fns)

$\int_a^b f(x)\,dx$ = a number!

e.g.

- $\int_1^2 x^2\,dx = \left|\begin{array}{c}\text{definite integral,}\\ \text{so a number}\end{array}\right| = \frac{1}{3}x^3\Big|_1^2$

$= \frac{1}{3}[8-1] = \frac{7}{3}$

- $\int x^2\,dx = \left|\begin{array}{c}\text{indefinite integral}\\ \text{so a function}\end{array}\right|$

$= \frac{1}{3}x^3 + C.$

This weekend's todo list: memorize the table of standard indef. integral (textbook)

e.g. $\int x^n\,dx = \frac{1}{n+1}x^{n+1}$, $n \neq -1$

$\int \sec x \tan x\,dx = \sec x + C$

$\int \sec^2 x\,dx = \tan x + C$

$\int \frac{dx}{1+x^2} = \tan^{-1}x + C.$

$\int^4 \sqrt{y} - y$

$\int \frac{dy}{y} = \ln|y| + C$

$$\int_1^4 \frac{\sqrt{y}-y}{y^2}\,dy =$$

$$\int \frac{dy}{y} = \ln|y| + c$$

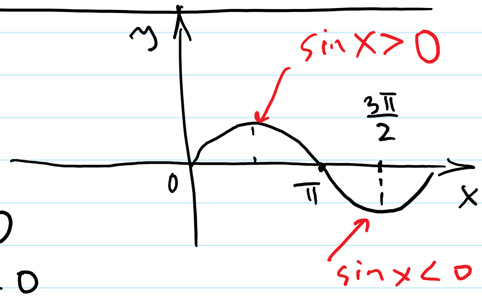
$$\int \frac{\sqrt{y}-y}{y^2}\,dy = \int \left(y^{-\frac{3}{2}} - y^{-1}\right)dy = \frac{1}{-\frac{3}{2}+1}\,y^{-\frac{3}{2}+1} - \ln|y| + c$$

$$= -2y^{-\frac{1}{2}} - \ln|y| + c$$

$$= -\frac{2}{\sqrt{y}} - \ln|y|\,\Big|_1^4 = (-1 - \ln 4) - (-2 - \overset{0}{\cancel{\ln 1}})$$

$$= -1 - \ln 4 + 2 = \underline{1} - \ln 4.$$

$$\int_0^{\frac{3\pi}{2}} |\sin x|\,dx$$



$$|\sin x| = \begin{cases} \sin x, & \text{if } \sin x \geq 0 \\ -\sin x, & \text{if } \sin x < 0 \end{cases}$$

So $|\sin x| = \sin x$ if $0 \leq x \leq \pi$

and $|\sin x| = -\sin x$ if $\pi < x \leq \frac{3\pi}{2}$

So $$\int_0^{\frac{3\pi}{2}} |\sin x|\,dx = \int_0^{\pi} |\sin x|\,dx + \int_{\pi}^{\frac{3\pi}{2}} |\sin x|\,dx$$

$$= \int_0^{\pi} \sin x\,dx - \int_{\pi}^{\frac{3\pi}{2}} \sin x\,dx = \ldots$$

(?) What is a marginal tax rate?

"A marginal tax rate is what you pay on the next dollar you earn."

$T(s)$ = amount of tax you pay on s dollars.

$M(s)$ = marginal tax at the level of

s dollars.

What is the connection b/w $T(s)$ and $M(s)$?

$$\boxed{T'(s) = M(s)}$$

$$M(s) \approx \frac{T(s+h) - T(s)}{h}$$

h is small

$$\int_0^{s_0} M(s)\,ds \overset{\text{FTC}}{=} T(s_0) - \underbrace{T(0)}_{=0} = T(s_0)$$