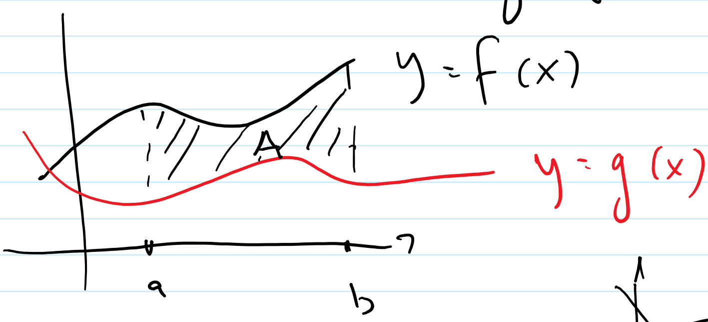
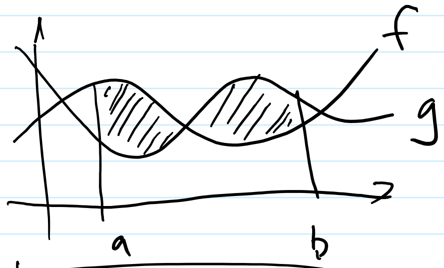
Area between graphs



$$\boxed{A = \int_a^b \left(f(x) - g(x)\right) dx}$$

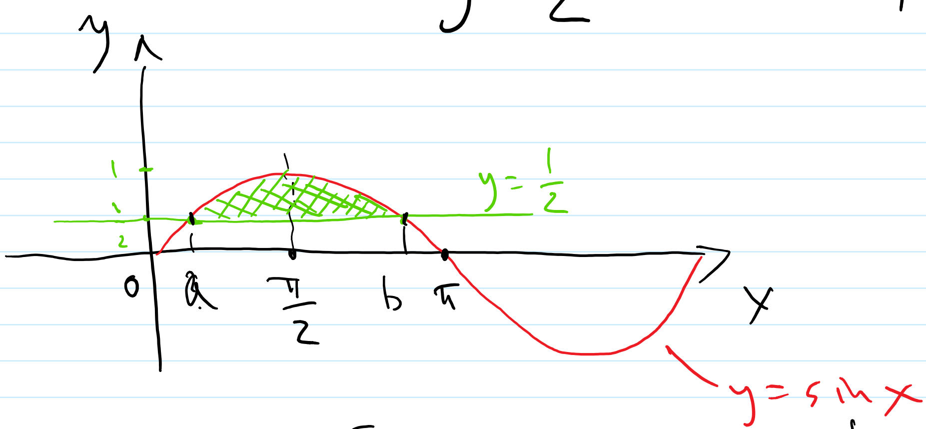
More generally,



$$\boxed{\text{Area} = \int_a^b |f-g|\, dx}$$

Examples

(1) Find the area between $y = \sin x$ and $y = \frac{1}{2}$ on $[0, 2\pi]$



a and b are solutions of $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\text{Area} = \int_{\pi/6}^{5\pi/6} \left(\sin x - \frac{1}{2}\right) dx \overset{\text{symmetry}}{=} 2 \int_{\pi/6}^{\pi/2} \left(\sin x - \frac{1}{2}\right) dx$$

$$= 2\left[-\cos x - \frac{1}{2}x\right]\Bigg|_{\pi/6}^{\pi/2} = \frac{\sqrt{3}}{2}$$

$$\Big[\ \cdots\ \Big]\Big|_{\frac{\pi}{6}}$$

$$= 2\cdot\left(-\frac{\pi}{4}\right) - 2\left(-\cos\frac{\pi}{6} - \frac{\pi}{12}\right) \qquad \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$= -\frac{\pi}{2} + \sqrt{3} + \frac{\pi}{6} = \sqrt{3} - \frac{\pi}{3}.$$

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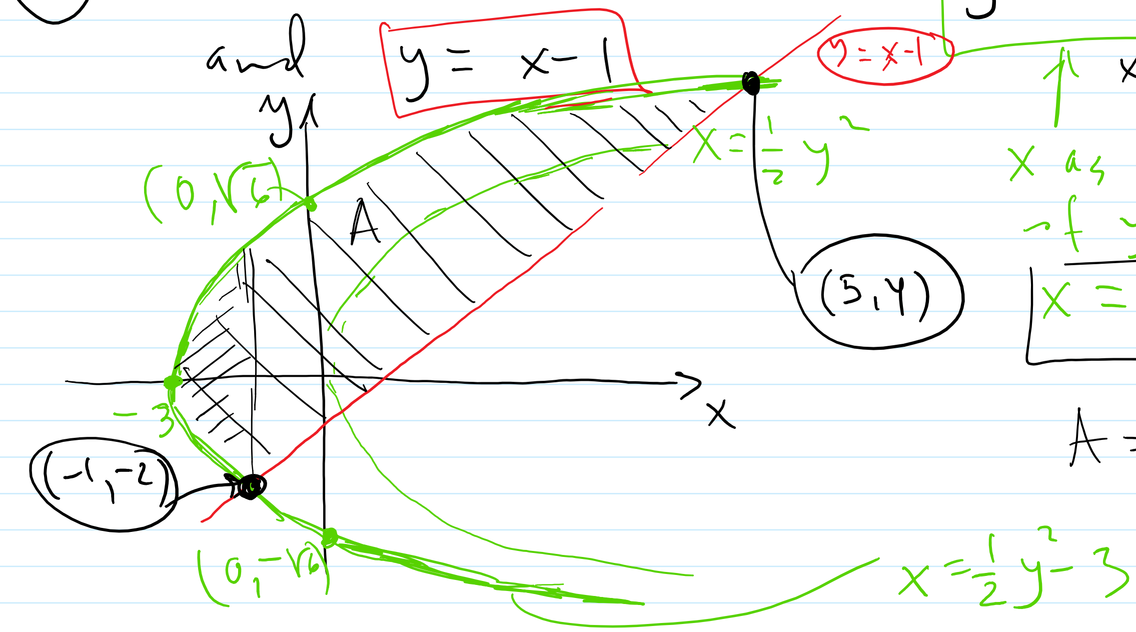
(2) Find the area between $\boxed{y^2 = 2x+6}$ and $\boxed{y = x-1}$



x as a function of y:

$$\boxed{x = \frac{y^2-6}{2} = \frac{1}{2}y^2 - 3}$$

$A = ?$

Points of intersection of $y^2 = 2x+6$ and $x = y+1 \Leftarrow y = x-1 \Rightarrow (x-1)^2 = 2x+6 \Rightarrow$

$$x^2 - 2x + 1 = 2x + 6$$
$$x^2 - 4x - 5 = 0$$
$$(x+1)(x-5) = 0$$

So points are:

$(-1, -2)$

$(5, 4)$

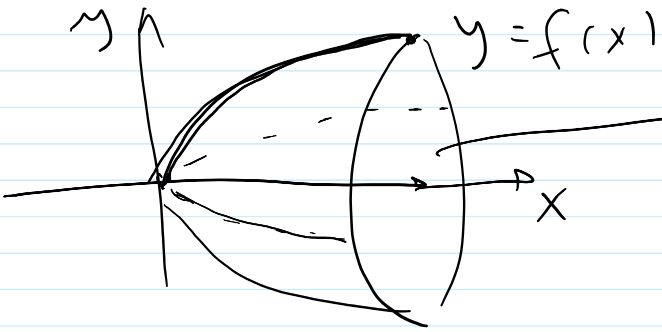
We treat x as a function of y. Then

$$\text{Area} = \int_{-2}^{4} \left[(y+1) - \left(\frac{1}{2}y^2 - 3\right)\right] dy$$

$$= \int_{-2}^{4} \left(-\frac{1}{2}y^2 + y + 4\right) dy$$

$$= \int_{-2} \left(-\frac{1}{2}y^2 + y + 4\right) dy$$

$$= -\frac{1}{6}y^3 + \frac{1}{2}y^2 + 4y \Big|_{-2}^{4} = \ldots$$



volume = ?