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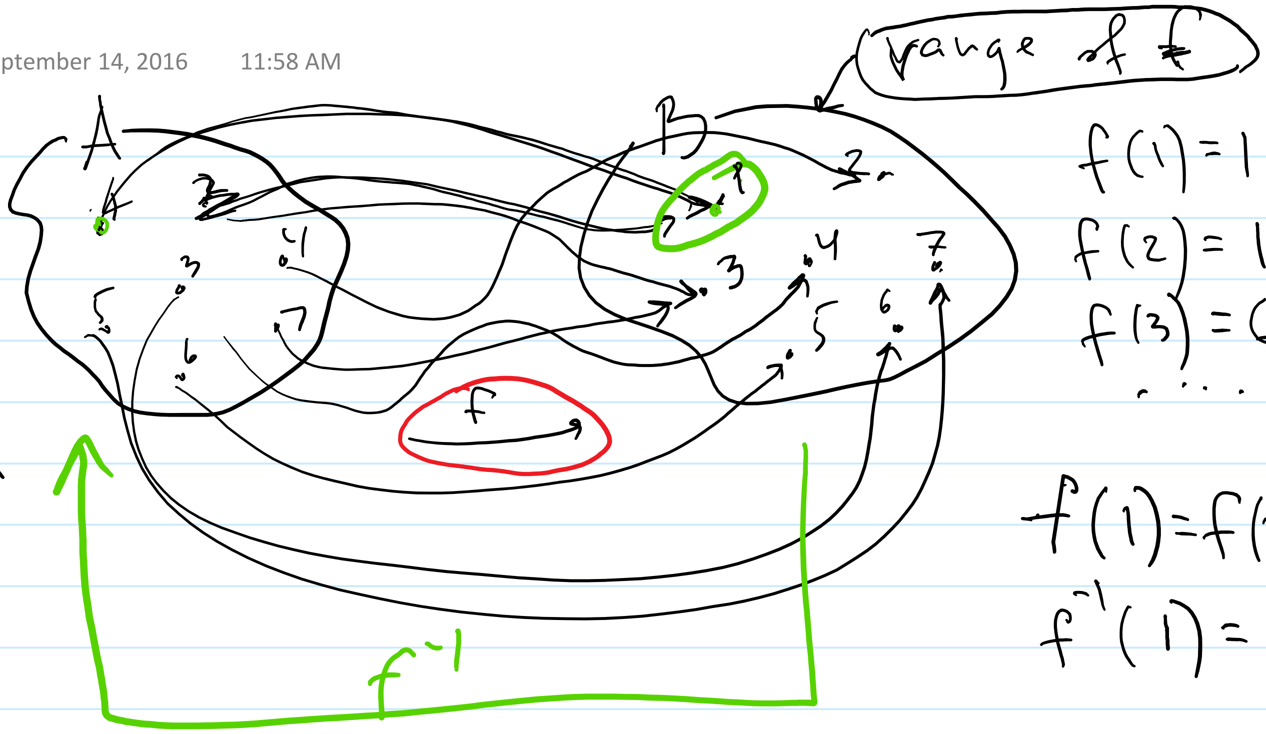


$A =$ domain of f

$f(1) = 1$

$f(2) = 1$

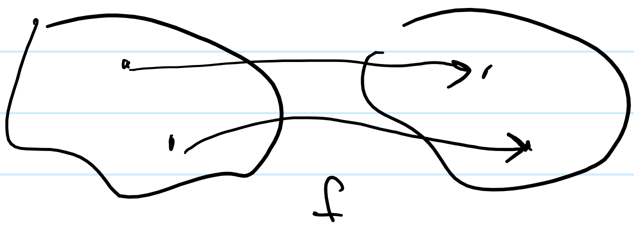
$f(3) = 6$

...

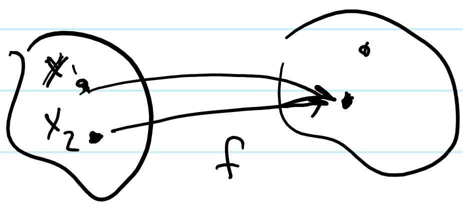
$f(1) = f(2) = 1$

$f^{-1}(1) = ?$

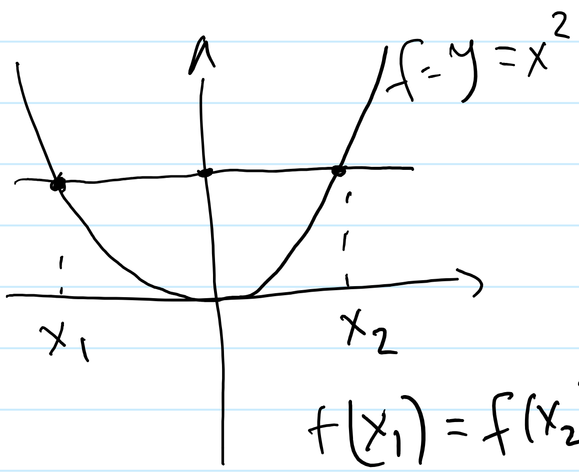
Def: A function $f: A \to B$ is called one-to-one (1-1) if $\boxed{f(x_1) \neq f(x_2)}$ for $\boxed{x_1 \neq x_2}$.



f is 1-1

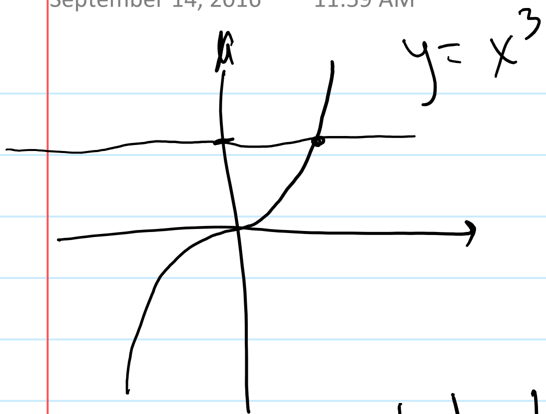


f is not 1-1.



← not a 1-1 function

$f(x_1) = f(x_2)$



This is a 1-1 function

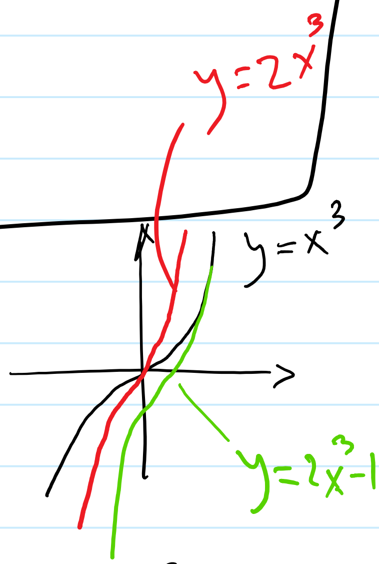
<u>Horizontal line test</u>: if any horizontal line intersects the graph of a function f at most at 1 point, then f is 1-1.

<u>Def</u>: Let $f : A \to B$ be 1-1. (B = range of f). Then $f^{-1} : B \to A$ is the inverse function if

$$\forall x \in A \quad f^{-1}(f(x)) = x$$
$$\forall y \in B \quad f(f^{-1}(y)) = y$$

e.g. $y = f(x) = 2x^3 - 1$

f is 1-1, Domain $= \mathbb{R}$
Range $= \mathbb{R}$



$f^{-1}(x) = \sqrt[3]{\frac{x+1}{2}}$

$y = 2x^3 - 1$ Solve this for x $y^3 \times 5x^2$

$y + 1 = 2x^3 \Rightarrow x = \sqrt[3]{\frac{y+1}{2}}$

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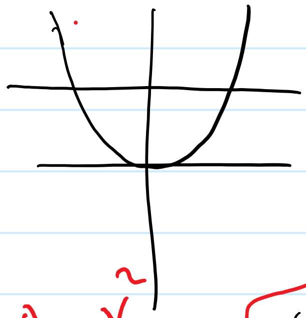
$y = x^2$

Domain $= \mathbb{R}$

Range $= \{ y \geq 0,\ y \in \mathbb{R} \} = [0, \infty)$

not 1-1



May restrict the domain

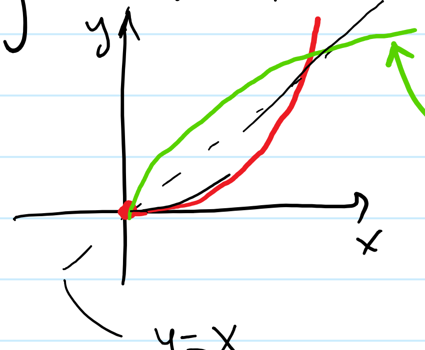
$f(x) = x^2$

$y = x^2$

$x \xrightarrow{f} x^2$

$\boxed{x \to x^2}$

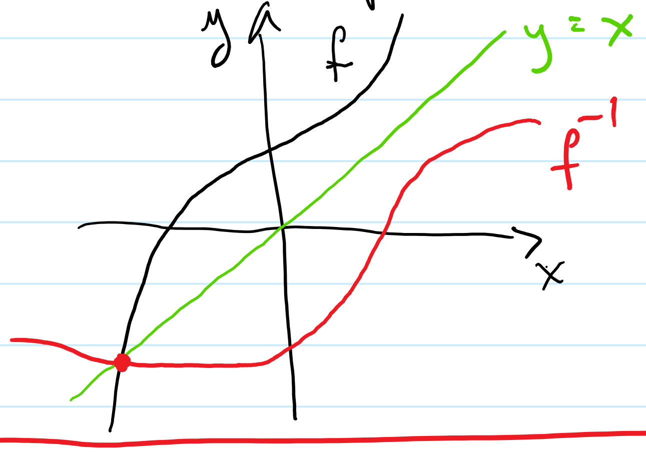
$x \geq 0$



inverse: $y = \sqrt{x}$

$y = x^2$

Rule: The graph of the inverse function is symmetric with the original function with respect to the line $\boxed{y = x}$



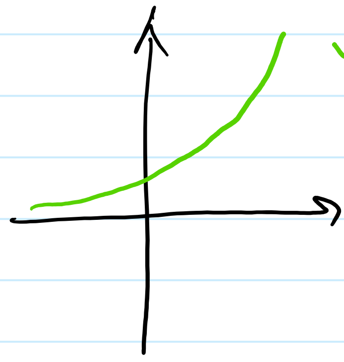
symmetry wrt x-axis

Suppose $y = b^x$

Domain $= \mathbb{R}$

$\boxed{\text{Range} = (0, \infty)}$

1-1 = yes



$y = b^x$

$b > 1$

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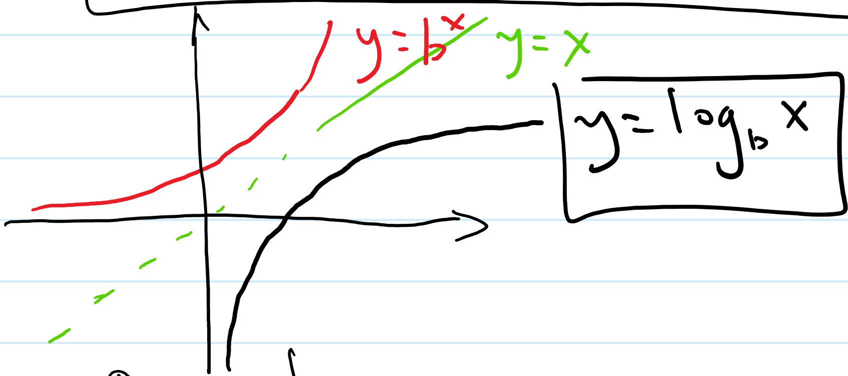
So $f: \boxed{y = b^x}$ has the inverse $\boxed{f^{-1} : (0,\infty) \to \mathbb{R}}$

$y = f^{-1} = \log_b x$. So

$$\boxed{\begin{array}{ll} \log_b b^x = x & \text{for all } x \in \mathbb{R} \\ \boxed{b^{\log_b x} = x} & \text{for all } \underline{x > 0} \end{array}} \leftarrow$$



from the symmetry.

Properties:

$$\log_b (xy) = \log_b x + \log_b y$$

$$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b (x^r) = r \log_b x$$

$y = e^x$ $\quad e = 2.71\ldots$

$\log_e x = \ln x$ "natural logarithm"

So: $\quad \ln e^x = x; \quad e^{\ln x} = x$

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Change of the base formula:

$$\boxed{\log_b X = \frac{\ln X}{\ln b}}$$

Pf: $y = \log_b X \Rightarrow b^y = X \xRightarrow{\ln(\cdot)}$

$\ln(b^y) = \ln X \Rightarrow y \cdot \ln b = \ln X$

$\boxed{y = \frac{\ln X}{\ln b}}$ Q.E.D.