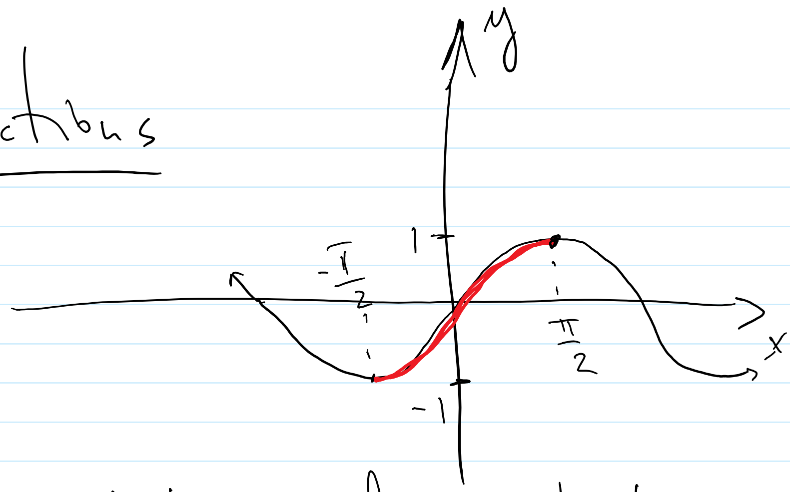
September 14, 2016 11:59 AM

Inverse Trig functions

$y = \sin x$ is not 1-1

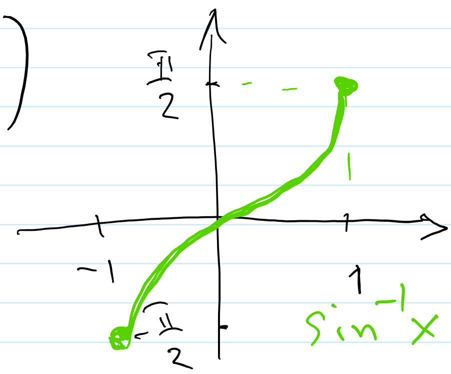


but $\begin{cases} y = \sin x \\ -\frac{\pi}{2} \le x \le \frac{\pi}{2} \end{cases}$ is 1-1 and so it has an inverse

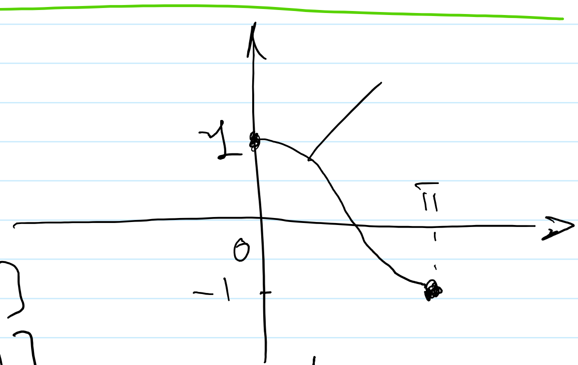
$y = \sin^{-1} x \quad (= (\sin x)^{-1})$

domain is $[-1, 1]$

range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

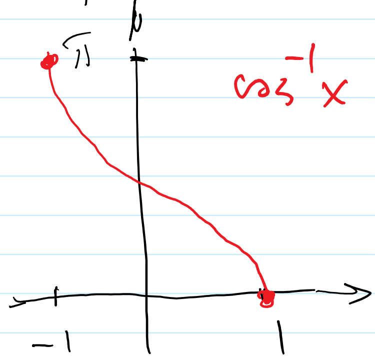


$\begin{cases} y = \cos x \\ 0 \le x \le \pi \end{cases}$



$y = \cos^{-1} x$ has domain $= [-1, 1]$

range $= [0, \pi]$



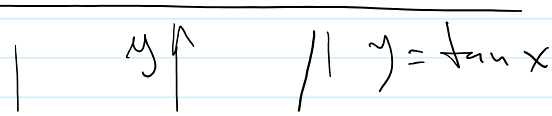
$\boxed{\sin^{-1}(\sin x) = x}$

true only for x in the range of $\sin^{-1}$, i.e.,

$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Similar for $\cos^{-1}$

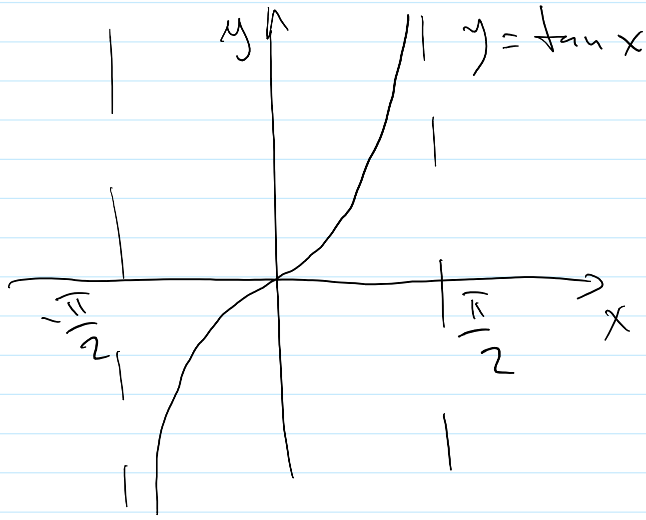
$y = \tan x$



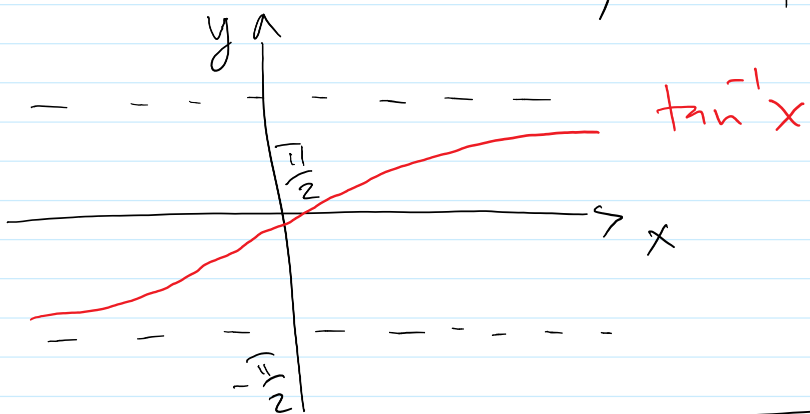
$y = \tan x$

is 1-1 on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

so there is an inverse:



$\tan^{-1} x : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Exercise: Simplify $\boxed{\sin(\tan^{-1} x)}$

① $\sqrt{1+x^2}$, x, θ, 1

$\boxed{\theta = \tan^{-1} x}$

$= \tan^{-1} \frac{x}{1}$

$\tan \theta = x$

$$\boxed{\sin \theta = \frac{x}{\sqrt{1+x^2}}}$$

So $\underbrace{\sin(\tan^{-1} x)}_{= \theta} = \frac{x}{\sqrt{1+x^2}}$

② $\tan^{-1} x = \theta$

$\tan \theta = x \implies$

$\frac{\sin \theta}{\cos \theta} = x \implies$

$\sin \theta = x \cos \theta \implies$

$\sin \theta = x \cdot \sqrt{1 - \sin^2 \theta}$

$\overset{2}{\implies} \sin^2 \theta = x^2 (1 - \sin^2 \theta)$

$\implies \sin^2 \theta + x^2 \sin^2 \theta = x^2$

$\sin^2 \theta = \frac{x^2}{1+x^2}$

$$\boxed{\sin \theta = \pm \frac{x}{\sqrt{1+x^2}}}$$

Question:

Why we get different answers?

different answers?