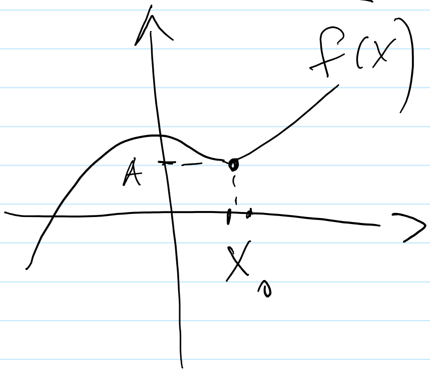
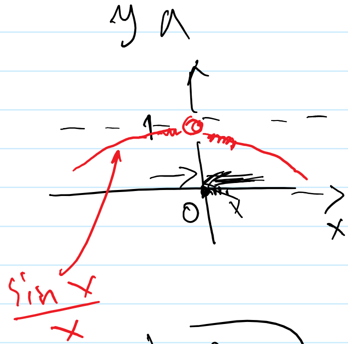
September 19, 2016 1:47 PM

Limits



When x approaches x_0, $f(x)$ approaches A.
The closer x is to x_0, the closer $f(x)$ is to A.

We write $\boxed{\lim_{x\to x_0} f(x) = A.}$

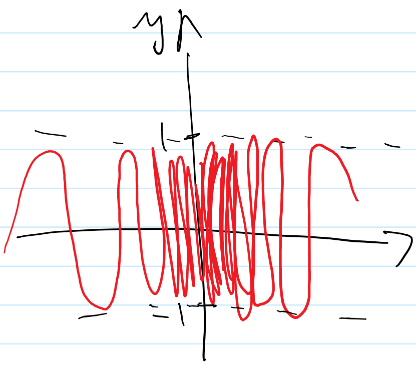


example:

① $\lim_{x\to 0} x^2 = 0.$

② $\lim_{x\to 0} \boxed{\frac{\sin x}{x}} = 1.$

$\boxed{\frac{\sin x}{x}}$ domain $\mathbb{R}\setminus\{0\}$



③ $\lim_{x\to 0} \sin\frac{1}{x}$ $\nexists$

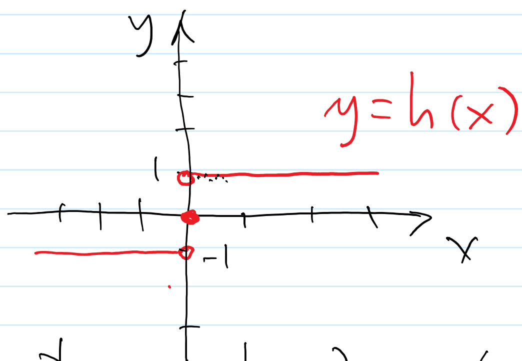
$-1 \le \sin\left(\frac{1}{x}\right) \le 1$

Try: $-0.01 < x < 0.01$

limit does not exist ($\nexists$)

One-sided limit

$$h(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$



- if $x \to 0$ from the left, then $h(x) \to -1$

$$\lim_{x\to 0^-} h(x) = -1$$

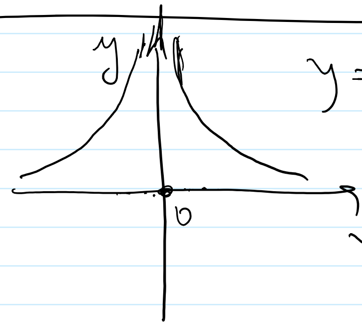
• if $x \to 0$ from the right, then $h(x) \to 1$.

$$\lim_{x \to 0^+} h(x) = 1.$$

Note: • both limits are different from $h(0) = 0$.

• $\lim_{x \to 0} h(x)$ does not exist.

Infinite limits



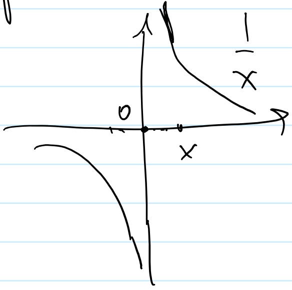
$y = \frac{1}{x^2}$

$\lim_{x \to a} f(x) = \infty$

if $f(x)$ becomes bigger and bigger as x approaches a.

e.g. $\boxed{\lim_{x \to 0} \frac{1}{x^2} = \infty}$

$y = \frac{1}{x}$



$$\lim_{x \to 0^+} \frac{1}{x} = \infty$$

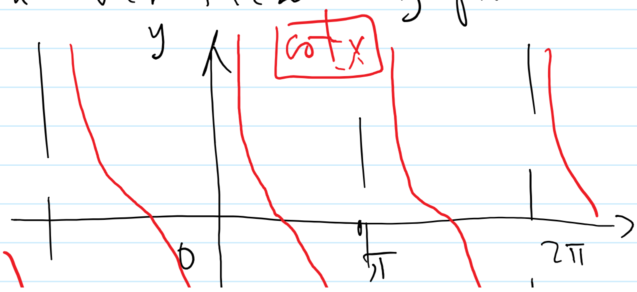
$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x \to 0} \frac{1}{x} \nexists.$$

Vertical asymptotes

if $\lim_{x \to a^+} f(x) = \infty$ or $-\infty$, $\lim_{x \to a^-} f(x) = \infty$ or $-\infty$

Then $f(x)$ has a vertical asymptote $\boxed{x = a}$

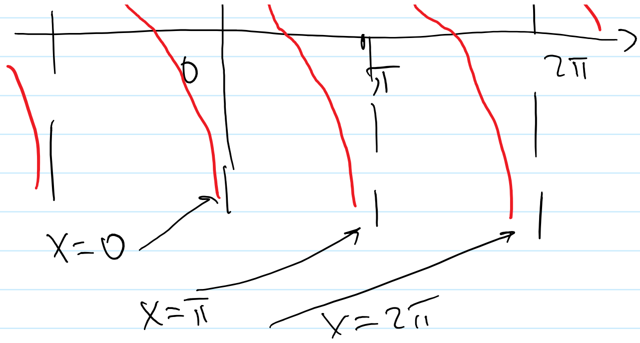


e.g. $y = \cot x$

$= \frac{\cos x}{}$

$$= \frac{\cos x}{\sin x}$$

vertical asymptotes

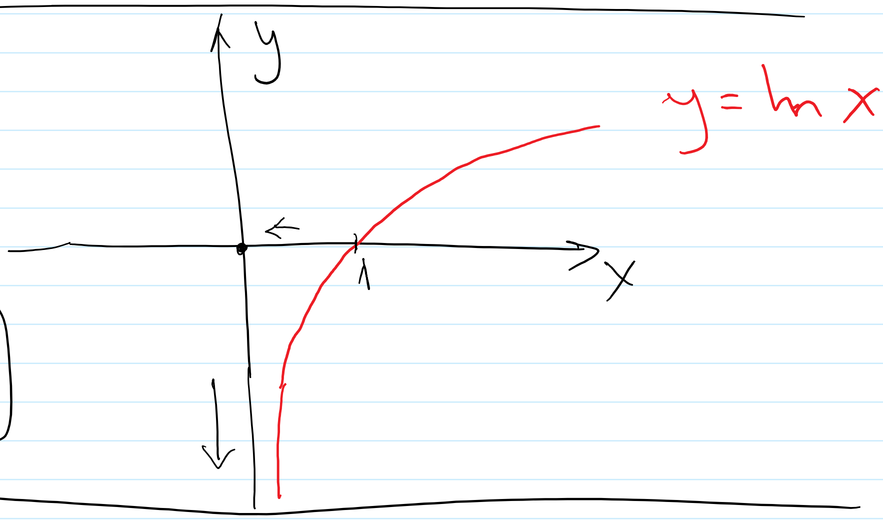


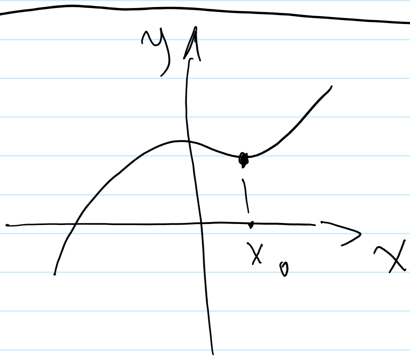
$\lim_{x \to 0} \cot x$ ∄ $\quad \lim_{x \to 0^+} \cot x = \infty$ etc.

$y = \ln x$

$x = 0$ – vertical asymptote,

$$\boxed{\lim_{x \to 0} \ln x = -\infty}$$





$$\lim_{x \to x_0} f(x) = f(x_0)$$

f is "continuous at x_0"