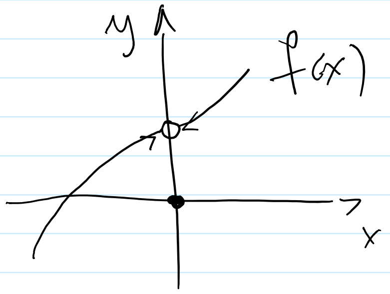
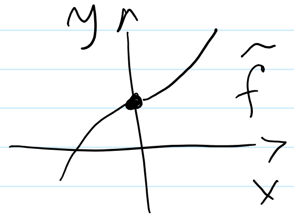
Def: A function $f(x)$ is continuous at a point x_0 if $\boxed{\lim_{x \to x_0} f(x) = f(x_0)}$.

A function is continuous on $[a,b]$ if it is continuous at every point of $[a,b]$.



f is not continuous at 0.



$\tilde{f}$ is continuous at 0.

All poly's, exp. fn, logs, trig functions are continuous on their domain

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e.g.

$$\lim_{x \to 1} e^{\left(\frac{1-\sqrt{x}}{1-x}\right)} = \left| \begin{array}{c} e^x \text{ is continuous} \\ \text{for } x \end{array} \right|$$

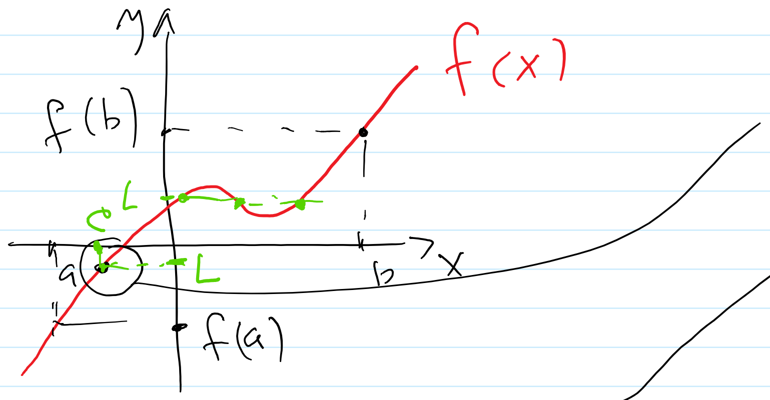
$$= e^{\lim_{x \to 1}\left(\frac{1-\sqrt{x}}{1-x}\right)} = e^{\frac{1}{2}}$$

$$\left\{ \begin{array}{l} \lim_{x \to 1} \frac{1-\sqrt{x}}{1-x} = \left| \frac{0}{0} \right| = \lim_{x \to 1} \frac{(1-\sqrt{x})(1+\sqrt{x})}{(1-x)(1+\sqrt{x})} \\ = \lim_{x \to 1} \frac{\cancel{1-x}}{\cancel{(1-x)}(1+\sqrt{x})} = \lim_{x \to 1} \frac{1}{1+\sqrt{x}} = \frac{1}{2} \end{array} \right.$$

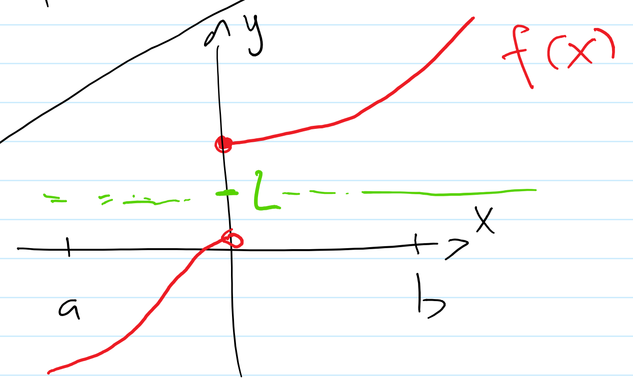
Thm (Intermediate Value Theorem):

Suppose $f(x)$ is a continuous function

Suppose $f(x)$ is a continuous function on $[a,b]$, $\boxed{f(a) < f(b)}$. Then for any $L \in (f(a), f(b))$ there exists a point $c \in (a,b)$ s.t. $f(c) = L$.



$f(c) = L$.



f is defined on $[a,b]$ but is discontinuous at $0 \in (a,b)$. For L as in the figure, there is no $c \in (a,b)$ s.t. $f(c) = L$.

Example: $p(x) = x^3 - 4x^2 + 20x - 17$ ||

Show that $\exists\, c \in \mathbb{R}$ s.t. $p(x) = 0$.

Solution:

$a = -10$ $\quad$ $p(-10) = -1000 + \ldots < 0$

$b = 10$ $\quad$ $p(10) = 1000 + \ldots > 0$

$p(x)$ is continuous on $[-10, 10]$ so for any $L \in (p(-10), p(10))$ $\exists\, c$ s.t. $p(c) = L$.

Take $L = 0 \in (p(-10), p(10))$

By the IVT $\exists\, c \in (-10, 10)$ s.t. $p(c) = 0$

$\square$

In fact, take any polynomial $= Q(x)$ of odd degree

Then for some large $A > 0$,

$$Q(A) > 0, \quad Q(-A) < 0.$$

$$x^7 + \dots \qquad A^7 + \dots > 0$$

$$(-A)^7 + \dots < 0$$

By the IVT $Q(x)$ has a root.

$$x^2 + 1 \neq 0$$