Name Solutions ID

UWO Calculus 1000 Quiz 10
December 2, 2016

(1) Let

$$F(x) = \int_1^x \left(\frac{t}{2} - \frac{2}{t}\right) dt$$

Find $F(2)$.

$$F(2) = \int_1^2 \left(\frac{t}{2} - \frac{2}{t}\right) dt = \frac{1}{2}\int_1^2 t\,dt - 2\int_1^2 \frac{1}{t}\,dt =$$

$$= \frac{t^2}{4}\Big|_1^2 - 2\ln|t|\Big|_1^2 = \frac{t^2}{4}\Big|_1^2 - 2\ln t\Big|_1^2 =$$

Answers:
(A) $\frac{3}{4} - \ln 4$
(B) 0
(C) $3 - 2\ln 2$
(D) $\frac{3}{4} - \ln 3$
(E) None of these

$$= \frac{2^2}{4} - \frac{1^2}{4} - 2\ln 2 + \underbrace{2\ln 1}_{=0} =$$

$$= \frac{3}{4} - 2\ln 2 = \frac{3}{4} - \ln 4.$$

(2) Evaluate

$$\int_0^1 xe^{-x^2} dx.$$

Put $u = -x^2$; then $du = -2x\,dx$, so

$$\int_0^1 xe^{-x^2} dx = -\frac{1}{2}\int_{u(0)}^{u(1)} e^u du = -\frac{1}{2}\int_0^{-1} e^u du =$$

Answers:
(A) $\frac{1}{2} - \frac{e}{2}$
(B) $1 - e$
(C) $1 - e^{-1}$
(D) $\frac{e-1}{2e}$
(E) None of these

$$= \frac{1}{2}\int_{-1}^{0} e^u du = \frac{1}{2}e^u\Big|_{-1}^{0} = \frac{1}{2}e^0 - \frac{1}{2}e^{-1} =$$

$$= \frac{1}{2} - \frac{1}{2e} = \frac{e-1}{2e}.$$

(3) An oil storage tank ruptures at time $t = 0$ and oil leaks from the tank at a rate of

$$r(t) = 100e^{-0.01t}$$

liters per minute. How much oil leaks out during the first hour?

We need to determine how much oil leaks during the first hour, or equivalently, during the first 60 minutes. As per the net change theorem, we need to compute $I = \int_0^{60} r(t)\,dt =$

$= 100 \int_0^{60} e^{-0.01t}\,dt$. Let $u = -0.01t$, hence $du = -0.01\,dt$. Substituting back in I, we get $I = -\frac{100}{0.01} \int_{u(0)}^{u(60)} e^u\,du = -10^4 \int_0^{-0.6} e^u\,du =$

$= 10^4 \int_{-0.6}^{0} e^u\,du = 10^4\, e^u \Big|_{-0.6}^{0} = 10^4\left(e^0 - e^{-0.6}\right) =$

$= 10^4\left(1 - e^{-0.6}\right)$, so the correct answer is (E).

Answers:

(A) $10000(e^{-0.01} - 1)$

(B) $(1 - e^{-0.01})10^4$

(C) $100(1 - e^{-0.01})$

(D) $e^{-0.01}$

(E) None of the above

Note: If the question gave the rate in litres/hr or, if the request were to find the amount of leaked oil during the first minute (keeping the rate at litres/minute) then we would need to compute $I = \int_0^1 r(t)\,dt$, which, as above, would give $I = 10^4(1 - e^{-0.01})$, hence (B) would be the correct answer.

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