

Solutions

Name _____

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UWO Calculus 1000 Quiz 3 Sept 30, 2016.

(1) Which of the following is true about the function

$$f(x) = \begin{cases} |x|, & \text{if } x < 1, \\ 1, & \text{if } x = 1, \\ x^2, & \text{if } x > 1. \end{cases}$$

on $(1, \infty)$, because x^2 is. Lastly, $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} |x| = 1 = f(1)$ and $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} x^2 = 1 = f(1)$, hence f is continuous on \mathbb{R} .

Answers:

- (A) f is continuous on \mathbb{R} .
(B) $f(x)$ is discontinuous at $x = 1$ only.
(C) $f(x)$ is discontinuous at $x = 0$ only.
(D) $f(x)$ is discontinuous at $x = 0$ and $x = 1$.
(E) None of these

(2) Compute

$$\lim_{x \rightarrow 0^+} \frac{x}{\ln x}.$$

We see that $\lim_{x \rightarrow 0^+} x = 0$ and $\lim_{x \rightarrow 0^+} \ln x = -\infty$,

$$\text{hence } \lim_{x \rightarrow 0^+} \frac{x}{\ln x} = \frac{0}{-\infty} = 0.$$

Answers:

- (A) ∞
(B) $-\infty$
(C) 0
(D) Limit does not exist
(E) None of these

(3) Evaluate the limit

$$\lim_{x \rightarrow \infty} [\ln(1+x^2) - \ln(1+x)].$$

We first note that if we take the limit of each term we end up with the special case $\infty - \infty$, therefore we need to find an alternative way to compute the limit.

Observe that $\ln(1+x^2) - \ln(1+x) =$
 $= \ln \frac{1+x^2}{1+x} = \ln \frac{x(\frac{1}{x} + x)}{x(\frac{1}{x} + 1)} = \ln \frac{\frac{1}{x} + x}{\frac{1}{x} + 1}.$

Since $\lim_{x \rightarrow \infty} (\frac{1}{x} + x) = \infty$ and $\lim_{x \rightarrow \infty} (\frac{1}{x} + 1) = 1$,

we have $\lim_{x \rightarrow \infty} \frac{\frac{1}{x} + x}{\frac{1}{x} + 1} = \infty$, hence

$$\lim_{x \rightarrow \infty} \ln \left(\frac{\frac{1}{x} + x}{\frac{1}{x} + 1} \right) = \infty.$$

Answers:

(A) 0

(B) 1

(C) $-\infty$ (D) ∞

(E) None of the above