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UWO Calculus 1000 Quiz 4
October 6, 2016

(1) Let

$$f(x) = 2^x.$$

Then the limit representing $f'(1)$ is

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} =$$

$$= \lim_{h\to 0} \frac{2^{1+h} - 2}{h} =$$

$$= \lim_{h\to 0} \frac{2(2^h - 1)}{h}.$$

Answers:

(A) $\lim_{h\to 0} \frac{2(2^h-1)}{h}$

(B) $\lim_{h\to 0} \frac{2^h-2}{h}$

(C) $\lim_{h\to 0} \frac{2^{1+h}-1}{h}$

(D) $\lim_{h\to 0} \frac{2^h-1}{h}$

(E) None of these

(2) Find the equation of the tangent line to the graph of the function $f(x)$ below at a point $x = 1$.

$$f(x) = x^2 - 3x.$$

Note that $f(1) = -2$, so we need to find the equation of the tangent line at $(1,-2)$. The slope $m = f'(1) = \lim_{h\to 0} \frac{(1+h)^2 - 3(1+h) + 2}{h} = \lim_{h\to 0} \frac{h^2 - h}{h} = \lim_{h\to 0} (h-1) = -1$. The general form of the equation of a line is $y = mx + n$. But the line passes through $(1,-2)$. So $-2 = (-1)1 + n$, i.e. $n = -1$.

The equation is $\boxed{y = -x - 1}$

Answers:

(A) $y = -x + 1$

(B) $y = -x + 3$

(C) $y = -x - 1$

(D) $y = x + 3$

(E) None of these

(3) The following limit represents the derivative of a function $g(x)$.

$$\lim_{h\to 0} \frac{\frac{1}{(2+h)^2} - \frac{1}{4}}{h}$$

Find the formula for $g(x)$ and the point at which the derivative is taken.

Let $g(x) = \frac{1}{x^2}$. Then

$$\lim_{h\to 0} \frac{\frac{1}{(2+h)^2} - \frac{1}{4}}{h} = \lim_{h\to 0} \frac{g(2+h) - g(2)}{h},$$

from which we see that the g defined above is the function we were looking for, and the point at which the derivative is taken is 2.

Answers:

(A) $g(x) = \dfrac{1}{x}$ and the derivative is taken at $x = 2$

(B) $g(x) = \dfrac{1}{x}$ and the derivative is taken at $x = \frac{1}{2}$

(C) $g(x) = \dfrac{1}{x^2}$ and the derivative is taken at $x = 2$

(D) $g(x) = \dfrac{1}{x^2}$ and the derivative is taken at $x = \frac{1}{2}$

(E) None of the above

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