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UWO Calculus 1000 Quiz 5
October 14, 2016

(1) Evaluate $g''\left(\frac{\pi}{3}\right)$, where $g(x) = \tan x$.

$g'(x) = \tan'(x) = \sec^2(x)$; $g''(x) = (g')'(x) =$
$= (\sec^2)'(x) \overset{\text{Chain Rule}}{=} 2\sec(x)\,[\sec(x)\tan(x)] =$
$= 2\sec^2(x)\tan(x)$. Since $\sec\left(\frac{\pi}{3}\right) = 2$ and
$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$, we obtain
$g''\left(\frac{\pi}{3}\right) = 2 \cdot 2^2 \cdot \sqrt{3} = 8\sqrt{3}$.

Answers:

(A) $g''\left(\frac{\pi}{3}\right) = 16\sqrt{3}$
(B) $g''\left(\frac{\pi}{3}\right) = 8\sqrt{3}$.
(C) $g''\left(\frac{\pi}{3}\right) = -8\sqrt{3}$
(D) $g''\left(\frac{\pi}{3}\right) = -4\sqrt{3}$
(E) None of these

(2) Evaluate the limit

$$\lim_{x\to 1} \frac{\sin(\sqrt{x}-1)}{x-1}.$$

Note that $x-1 = (\sqrt{x}-1)(\sqrt{x}+1)$, hence $\lim\limits_{x\to 1} \frac{\sin(\sqrt{x}-1)}{x-1} =$
$= \lim\limits_{x\to 1}\left[\frac{\sin(\sqrt{x}-1)}{\sqrt{x}-1} \cdot \frac{1}{\sqrt{x}+1}\right]$. Denoting $\theta = \sqrt{x}-1$,
we have $\theta \to 0$ when $x \to 1$, so $\lim\limits_{x\to 1} \frac{\sin(\sqrt{x}-1)}{\sqrt{x}-1} =$
$= \lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta} = \boxed{1.}$ Also, $\lim\limits_{x\to 1} \frac{1}{\sqrt{x}+1} = \boxed{1/2}$

Since both limits exist, we have
$\lim\limits_{x\to 1}\left[\frac{\sin(\sqrt{x}-1)}{\sqrt{x}-1} \cdot \frac{1}{\sqrt{x}+1}\right] = \lim\limits_{x\to 1}\left(\frac{\sin(\sqrt{x}-1)}{\sqrt{x}-1}\right) \cdot$
$\cdot \lim\limits_{x\to} \frac{1}{\sqrt{x}+1} = 1 \cdot \frac{1}{2} = \frac{1}{2}$.

Answers:

(A) 1
(B) 2
(C) $\frac{1}{2}$
(D) Limit does not exist
(E) None of the above

(3) Find equation of the tangent line to the graph of the function below at $x = 0$.

$$y = e^{x+\cos x}.$$

We need to find the equation of a line $y = mx + n$, where m is the slope of the line and n gives the y-axis intercept. Since our line is tangent to the function above, at 0, we have $m = y'(0)$, so we first compute the derivative:

$$y'(x) = \left(e^{x+\cos x}\right)' \quad \underline{\underline{\text{chain rule}}}$$

$$= e^{x+\cos x}\left(x+\cos x\right)' = e^{x+\cos x}(1-\sin x).$$

Answers:

(A) $y = x + e$

(B) $y = ex + 2e$

(C) $y = 2ex + e$

(D) $y = ex + e$

(E) None of these

So, $m = y'(0) = e^{0+\cos 0}(1-\sin 0) = \boxed{e}$

Also, for $x=0$ we get $n = y(0) = e^{0+\cos 0} = \boxed{e}$

Therefore, the equation of the line we were looking for is given by

$$\boxed{y = ex + e}.$$

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