

# Solutions (Quiz Version 1)

Name \_\_\_\_\_

ID \_\_\_\_\_

## UWO Calculus 1000 Quiz 6 November 3, 2016

- (1) The radius of a sphere is increasing at a rate 2 m/sec. How fast is the volume increasing when the radius is 10 m?

Let  $r(t)$  be the radius at time  $t$ , so  $r'(t) = 2$ .

The volume  $V(t) = \frac{4\pi}{3} r^3(t)$ , hence

$V'(t) = \frac{4\pi}{3} \cdot 3r^2(t) r'(t)$ . If at some time  $t_0$ ,

$r(t_0) = 10$ , then  $V'(t_0) = 4\pi r^2(t_0) r'(t_0) =$

$$= 4\pi \cdot 100 \cdot 2 = 800\pi.$$

Answers:

(A)  $800\pi$

(B)  $400\pi$

(C)  $1200\pi$

(D)  $\frac{800}{3}\pi$

(E) None of these

- (2) Find local maxima and local minima of the function below on its domain.

$$F(x) = \frac{x^2}{x-1}$$

$$F'(x) = \frac{2x(x-1) - x^2}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}. \quad F'(x) = 0 \text{ if}$$

$x = 0$  or  $x = 2$ . On  $(-\infty, 0)$ ,  $F'(x) > 0$  hence

$F$  is increasing. On  $(0, 2)$   $F'(x) < 0$ , hence  $F$  is

decreasing there. On  $(2, \infty)$   $F'(x) > 0$ , hence

Answers:

(A)  $x = 0$  and  $x = 2$  are local minima

(B)  $x = 0$  is a local minimum, and  $x = 2$  is a local maximum

(C)  $x = 0$  is a local maximum, and  $x = 2$  is a local minimum

(D)  $x = 0$  is a local maximum, and  $x = 1$  is a local minimum

(E) None of the above

$F$  is increasing, so  $x = 0$  is a local maximum and  $x = 2$  is a local minimum.

(3) Which of the following is FALSE about the function given on the graph below?

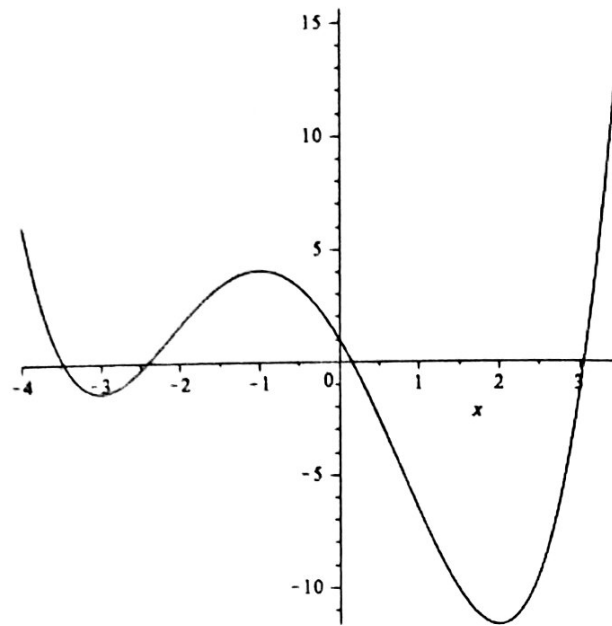


FIGURE 1. Graph of  $f(x)$

A is True: there are three local extreme values of  $f$  and no other critical points;

Answers:

- (A) The function  $f(x)$  has three critical points
- (B) The function  $f(x)$  has the global maximum at  $x = -1$
- (C) The function  $f(x)$  is decreasing on the interval  $(-1, 2)$
- (D)  $f''(2) > 0$
- (E) None of these

B is False:

e.g.  $f(-4) > f(-1)$

D - true:  $f$  is upward concave on an interval containing 2

E is clearly False

Mark: \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_ /15

Name Solution (Quiz Version 2)

ID \_\_\_\_\_

UWO Calculus 1000 Quiz 6  
November 3 2016

- (1) The radius of a sphere is increasing at a rate 3 m/sec. How fast is the volume increasing when the radius is 10 m?

Let  $r(t)$  be the radius at time  $t$ , so  $r'(t) = 3$ . The volume  $V(t) = \frac{4\pi}{3} r^3(t)$ , hence  $V'(t) = \frac{4\pi}{3} \cdot 3 r^2(t) r'(t)$ . If at some time  $t_0$ ,  $r(t_0) = 10$ , then  $V'(t_0) = 4\pi r^2(t_0) r'(t_0) = 4\pi \cdot 100 \cdot 3 = 1200\pi$ .

Answers:

- (A)  $800\pi$   
(B)  $1200\pi$   
(C)  $200\pi$   
(D)  $\frac{800}{3}\pi$   
(E) None of these

- (2) Find local maxima and local minima of the function below on its domain.

$$F(x) = \frac{x^2}{1-x}$$

$$F'(x) = \frac{2x(1-x) + x^2}{(1-x)^2} = \frac{x(2-x)}{(1-x)^2}, \quad F'(x) = 0 \text{ if}$$

$x=0$  or  $x=2$ . On  $(-\infty, 0)$   $F'(x) < 0$ , hence  $F$  is decreasing there. On  $(0, 2)$   $F'(x) > 0$ , hence  $F$  is increasing. On  $(2, \infty)$   $F'(x) < 0$ , hence  $F$  is decreasing. So  $x=0$  is a local minimum and  $x=2$  is a local maximum.

Answers:

- (A)  $x=0$  and  $x=2$  are local minima  
(B)  $x=0$  is a local minimum, and  $x=2$  is a local maximum  
(C)  $x=0$  is a local maximum, and  $x=2$  is a local minimum  
(D)  $x=0$  is a local maximum, and  $x=1$  is a local minimum  
(E) None of the above

(3) Which of the following is FALSE about the function given on the graph below?

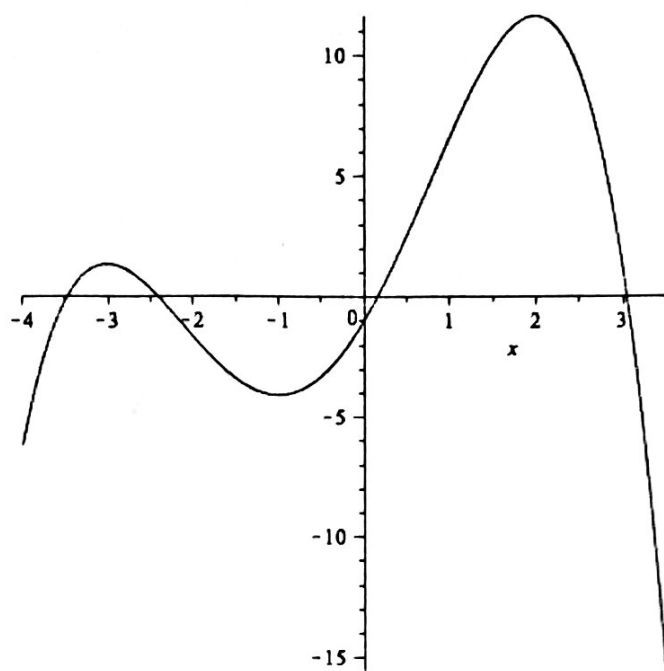


FIGURE 1. Graph of  $f(x)$

A is True; there are three local extreme values of  $f$  and no other critical points;

Answers:

- (A) The function  $f(x)$  has three critical points  
 (B) The function  $f(x)$  has the global maximum at  $x = 2$   
 (C) The function  $f(x)$  is increasing on the interval  $(-1, 2)$

- (D)  $f''(2) > 0$   
 (E) None of these

B is True; clear from the graph;

C is True; clear from the graph;

D is False;  $f$  is downward concave on an interval containing 2;

E is clearly False.