Solutions (Version 1)

Name ______ ID ______

UWO Calculus 1000 Quiz 7
November 11, 2016

(1) Evaluate

$$\lim_{x\to\infty}\frac{x^5}{e^x}.$$

We are in the indeterminate case $\frac{\infty}{\infty}$ and we can apply l'Hospital's rule:

$$\lim_{x\to\infty}\frac{x^5}{e^x} \overset{H}{=} \lim_{x\to\infty}\frac{5x^4}{e^x} \overset{H}{=} \lim_{x\to\infty}\frac{5\cdot 4x^3}{e^x} \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to\infty}\frac{5\cdot4\cdot3x^2}{e^x} \overset{H}{=} \lim_{x\to\infty}\frac{5\cdot4\cdot3\cdot2x}{e^x} \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to\infty}\frac{120}{e^x} = 0.$$

Answers:
(A) 0
(B) ∞
(C) 1
(D) 5!
(E) None of these

(2) Evaluate

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}.$$

We are in the indeterminate case 1^∞. We take first the $\ln\left[\left(1+\frac{2}{x}\right)^{3x}\right]$ and compute its limit:

$$\lim_{x\to\infty}\ln\left[\left(1+\frac{2}{x}\right)^{3x}\right] = \lim_{x\to\infty} 3x\ln\left(1+\frac{2}{x}\right) = \lim_{x\to\infty}\frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{3x}} \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to\infty}\frac{\frac{1}{1+\frac{2}{x}}\left(-\frac{2}{\cancel{x^2}}\right)}{-\frac{1}{3\cancel{x^2}}} = \lim_{x\to\infty}\frac{6}{1+\frac{2}{x}} = 6.$$

Hence, $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x} = \lim_{x\to\infty} e^{\ln\left[\left(1+\frac{2}{x}\right)^{3x}\right]} = e^6.$

Answers:
(A) 1
(B) e^2
(C) e^6
(D) ∞
(E) None of these

(3) The product of two positive numbers equals 9. Find the smallest possible value for the sum of squares of these numbers.

Let $x, y \in \mathbb{R}$ be positive such that $xy = 9$, hence none of them are equal to 0. This means that $y = \frac{9}{x}$. We need to find the minimum value of the sum $x^2 + y^2 = x^2 + \frac{81}{x^2}$. If we put $f(x) = x^2 + \frac{81}{x^2}$, defined on $(0, \infty)$, we need to find the absolute minimum value of f on $(0, \infty)$. Since f is differentiable on $(0, \infty)$, we look for the solutions of $f'(x) = 0$, that is

$f'(x) =$ $2x - 2\frac{81}{x^3} = 0$, or $x - \frac{81}{x^3} = 0$, i.e.

$x^4 - 81 = 0$, which leads to $x = 3$ (since $x > 0$, we ignore the other solution $x = -3$). So, $x = 3$ is the only critical point of f and, since

Answers:
(A) 9
(B) 18
(C) 12
(D) 3
(E) None of the above

$f''(x) = 2 + \frac{6 \cdot 81}{x^4} > 0$, in particula $f''(3) > 0$, it follows that $x = 3$ is a point of (absolute) minimum. The minim value of the sum is $f(3) = 3^2 + \frac{81}{3^2} = 9 + 9 = 18$.

Mark: ______ + ______ + ______ + = ______ /15

Solutions

ame (Version 2) ID

UWO Calculus 1000, Quiz 7
November 11, 2016

(1) Evaluate

$$\lim_{x\to\infty} \frac{e^x}{x^5}.$$

We are in the indeterminate case $\frac{\infty}{\infty}$ and we can apply l'Hospital's rule:

$$\lim_{x\to\infty} \frac{e^x}{x^5} \overset{H}{=} \lim_{x\to\infty} \frac{e^x}{5x^4} \overset{H}{=} \lim_{x\to\infty} \frac{e^x}{5\cdot 4x^3} \overset{H}{=} \lim_{x\to\infty} \frac{e^x}{5\cdot 4\cdot 3x^2} \overset{H}{=}$$

$$\overset{H}{=} \lim_{x\to\infty} \frac{e^x}{5\cdot 4\cdot 3\cdot 2x} \overset{H}{=} \lim_{x\to\infty} \frac{e^x}{120} = \infty.$$

Answers:

(A) 0

(B) ∞

(C) 1

(D) $\frac{1}{5!}$

(E) None of these

(2) Evaluate

$$\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^{3x}.$$

This is the same problem as in Quiz version 1 (see solution on page 1).

Answers:

(A) e^6

(B) 1

(C) e^2

(D) ∞

(E) None of these

(3) The product of two positive numbers equals 9. Find the smallest possible value for the sum of squares of these numbers.

This is the same problem as question 3 in Quiz version 1 (see solution on page 2).

Answers:

(A) 12

(B) 9

(C) 18

(D) 3

(E) None of the above