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UWO Calculus 1000 Quiz 8
November 18, 2016

(1) Find the most general antiderivative of the function

$$f(x) = \frac{5}{\sqrt{x}} + \sin 3x.$$

An antiderivative of $\frac{5}{\sqrt{x}} = 5^{-1/2}$ is given by $5\,\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = 5 \cdot 2x^{1/2} = 10\sqrt{x}$. An antiderivative of $\sin 3x$ is given by $-\frac{1}{3}\cos 3x$. Therefore the most general antiderivative of f is given by

$$10\sqrt{x} - \frac{1}{3}\cos 3x + C, \quad C \in \mathbb{R}.$$

Answers:
(A) $\frac{2}{5}\sqrt{x} - \frac{1}{3}\cos 3x + C$
(B) $\frac{2}{5}\sqrt{x} + \frac{1}{3}\cos 3x + C$
(C) $10\sqrt{x} - 3\cos 3x + C$
(D) $10\sqrt{x} - \frac{1}{3}\cos 3x + C$
(E) None of these

(2) Let $f(x)$ be the function such that $f(0) = 0$ and

$$f'(x) = \frac{x^2+2}{1+x^2}.$$

Find $f(1)$.

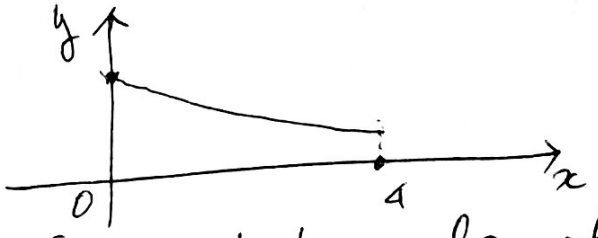
$$f'(x) = \frac{x^2+2}{1+x^2} = \frac{x^2+1+1}{x^2+1} = \frac{x^2+1}{x^2+1} + \frac{1}{x^2+1} = 1 + \frac{1}{x^2+1}.$$

So, f (as an antiderivative of f') is of the form $f(x) = x + \arctan x + C$, for some constant $C \in \mathbb{R}$. From $f(0) = 0$ we get $0 + \arctan 0 + C = 0$, i.e. $C = 0$, therefore $f(x) = x + \arctan x$. It follows that $f(1) = 1 + \frac{\pi}{4}$.

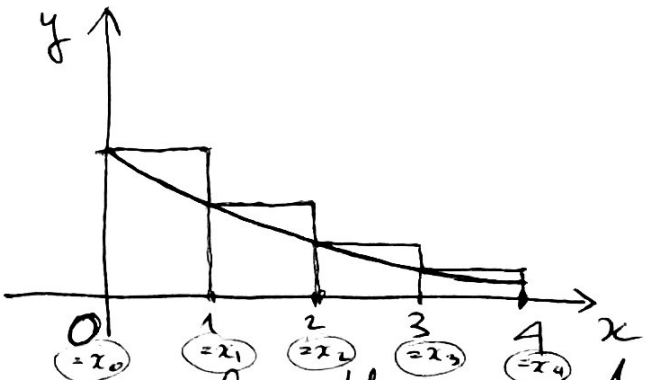
Answers:
(A) 1
(B) $\tan 1$
(C) $1 + \frac{\pi}{4}$
(D) $\frac{\pi}{4}$
(E) None of these

(3) Estimate the area under the graph of the function $y = \frac{1}{1+x}$ on the interval $[0, 4]$ using 4 intervals and the left endpoints.

The function $y = \frac{1}{1+x}$ is decreasing on $[0,4]$, so a sketch of its graph looks like this



Divide $[0,4]$ in four intervals of equal lengths, hence $\Delta x = \frac{4}{4} = 1$, and draw the rectangles given by the left endpoints:



Answers:

(A) $\frac{25}{12}$

(B) $\frac{67}{60}$

(C) $\frac{11}{6}$

(D) $\frac{5}{4}$

(E) None of the above

The estimated area under the graph of y is

$$A = \sum_{i=0}^{3} y(x_i)\,\underbrace{\Delta x}_{=1} = y(0)+y(1)+y(2)+y(3) =$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}. \checkmark$$

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