

Solutions

Name _____

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UWO Calculus 1000 Quiz 9

November 25, 2016

(1) Let

$$f(x) = \int_0^x \sqrt{1+t^2} dt.$$

Evaluate $\frac{d^2 f}{dx^2}(1)$.

By FTC (I), we have $\frac{df}{dx}(x) = \sqrt{1+x^2}$;
then, $\frac{d^2 f}{dx^2}(x) = \frac{d}{dx} \left(\frac{df}{dx}(x) \right) = \frac{d}{dx} (\sqrt{1+x^2}) =$
 $= \frac{1}{2\sqrt{1+x^2}} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$. So,

Answers:

(A) $\frac{\sqrt{2}}{2}$

(B) $\sqrt{2}$

(C) $\frac{1}{2\sqrt{2}}$

(D) 1

(E) None of these

$$\frac{d^2 f}{dx^2}(1) = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

(2) Evaluate

$$\int_0^{\pi/2} \cos 3\theta d\theta.$$

Since $\frac{1}{3} (\sin 3\theta)' = \frac{1}{3} \cdot 3 \cos 3\theta = \cos 3\theta$, we

get $\int_0^{\pi/2} \cos 3\theta d\theta = \int_0^{\pi/2} \frac{1}{3} (\sin 3\theta)' d\theta =$

Answers:

(A) 0

(B) -1

(C) $-\frac{1}{3}$

(D) $-\frac{3\pi}{2}$

(E) None of these

$$= \frac{1}{3} \int_0^{\pi/2} (\sin 3\theta)' d\theta \stackrel{\text{FTC (II)}}{=} \frac{1}{3} \sin 3\theta \Big|_0^{\pi/2} =$$
$$= \frac{1}{3} \left[\sin \left(\frac{3\pi}{2} \right) - \sin 0 \right] = -\frac{1}{3}.$$

(3) Evaluate the integral

$$\int_{-1}^2 (x + |x|) dx$$

Note that $|x| = \begin{cases} -x, & \text{if } -1 \leq x < 0 \\ x, & \text{if } 0 \leq x \leq 2 \end{cases}$.

$$\begin{aligned} \text{Therefore, } \int_{-1}^2 (x + |x|) dx &= \int_{-1}^0 (x + |x|) dx + \\ &+ \int_0^2 (x + |x|) dx = \int_{-1}^0 (x - x) dx + \int_0^2 (x + x) dx = \\ &= \underbrace{\int_{-1}^0 0 dx}_{=0} + \int_0^2 2x dx = 2 \int_0^2 x dx \quad \underline{\underline{\text{FTC(II)}}} \\ &= 2 \left. \frac{x^2}{2} \right|_0^2 = 2^2 - 0^2 = 4. \end{aligned}$$

Answers:

(A) 0

(B) 4

(C) 3

(D) 5

(E) None of the above