**Homework 4.**
Due November 14.

1. Textbook, Problem 7 on page 74.
2. Textbook, Problem 12 on page 74.
3. Textbook, Problem 1 on page 90.
4. Suppose that $f(z)$ is an entire function such that

$$\lim_{z\to\infty} f(z) = \infty.$$

   Prove that there exists a point $z_0 \in \mathbb{C}$ such that $f(z_0) = 0$.

5. (MATH 9024 only) Prove that the function

$$f(z) = \sum_{j=0}^{\infty} 2^{-j} z^{2^j}$$

   is holomorphic on the open unit disc $\mathbb{B}(0,1) = \{|z| < 1\}$ and continuous on its closure. Prove that if $w$ is a $2^N$-th root of unity, then

$$\lim_{r\to 1^-} |f'(rw)| = +\infty.$$

   Deduce that $f$ cannot be the restriction to $\mathbb{B}(0,1)$ of a holomorphic function defined on a connected open set that is strictly larger than $\mathbb{B}(0,1)$.