

Homework 5.

Due December 2.

1. Prove that if $f(z)$ is holomorphic in a punctured neighbourhood of z_0 and has a nonremovable singularity at z_0 , then the function $e^{f(z)}$ has an essential singularity at z_0 .
2. Textbook, Problem 5 on page 126.
3. Prove that the Sohotsky-Casorati-Weierstrass Theorem also holds for a singularity which is a limit of poles.
4. Textbook, Problem 2 on page 141.
5. Evaluate the following integral using the residue theory:

$$\int_{-\infty}^{\infty} \frac{x dx}{(x^2 + 4x + 13)^2}$$

6. (MATH 9024 only) Prove that if K_1 and K_2 are two disjoint compacts in \mathbb{C} and $f(z)$ is holomorphic on $\mathbb{C} \setminus (K_1 \cup K_2)$, then there exist holomorphic functions $f_j \in \mathcal{O}(\mathbb{C} \setminus K_j)$, $j = 1, 2$, such that $f = f_1 + f_2$.