COMPLEX ANALYSIS II, MATH 9056, FALL 2017

HOMEWORK ASSIGNMENT 1.

Due October 3, in class.

1.1. Find the number of zeros of the polynomial $p(z) = z^9 + 2z^5 - 2z^4 + z + 3$ in the right half-plane.

1.2. Suppose that $f(z)$ is holomorphic in the closure of the unit disc $\mathbb{D} = \{|z| \leq 1\}$ and $|f(z)| = \text{const}$ on $\partial\mathbb{D}$. Prove that either $f = \text{const}$, or $f(z)$ vanishes somewhere in $\mathbb{D}$.

1.3. Construct a sequence of nonunivalent holomorphic functions $\{f_n(z)\}$ in $\mathbb{D}$ which converges normally to a function $f(z)$ univalent on $\mathbb{D}$.

1.4. Suppose $f : \mathbb{D} \to \mathbb{D}$ is holomorphic, and

$$f(0) = f'(0) = \cdots = f^{(k-1)}(0) = 0.$$

Prove that $|f(z)| \leq |z|^k$, for all $z \in \mathbb{D}$.

1.5. Let $f(z)$ be meromorphic in the unit disc $\mathbb{D}$ and holomorphic in a neighbourhood of the unit circle $\partial\mathbb{D} = \{|z| = 1\}$. Prove that for any number A such that

$$|A| > \max_{z \in \partial\mathbb{D}} |f(z)|$$

the number of points in $\mathbb{D}$ where f attains value A equals the number of poles of f in $\mathbb{D}$.