

## COMPLEX ANALYSIS II, MATH 9056, FALL 2017

### HOMEWORK ASSIGNMENT 3.

Due Thursday, November 9.

- 3.1. Evaluate explicitly the infinitesimal form  $q_{\mathbb{H}}(z)$  of the Carathéodory metric on the upper half plane  $\mathbb{H} = \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$ .
- 3.2. Does there exist a holomorphic map from the unit disc onto  $\mathbb{C}$ ?
- 3.3. Find a conformal map from the strip

$$\{-2017\pi < \operatorname{Re} z < 2017\pi\}$$

onto the unit disc.

- 3.4. Suppose that  $\{f_{\alpha}\}$  is a family of holomorphic functions on a domain  $\Omega$  such that any sequence of functions in the family has a subsequence that converges normally on  $\Omega$ . Prove that the family  $\{f'_{\alpha}\}$  has the same property.
- 3.5. Show that the function  $z \rightarrow z^2$  is not a strict contraction of the hyperbolic disc, that is, show that there is no constant  $c < 1$  such that  $\rho(z^2, \zeta^2) \leq c\rho(z, \zeta)$  for all  $z, \zeta \in \mathbb{D}$ .