COMPLEX ANALYSIS II, MATH 9056, FALL 2017

HOMEWORK ASSIGNMENT 5.

Due December 5

5.1. Let $\{z_k\}$ be a sequence of distinct points in a domain Ω that accumulates on $\partial\Omega$. Let $\{m_k\}$ be a sequence of positive integers, and for each k, let $a_{k0}, \dots, a_{km_k}$ be complex numbers. Show that there exists a holomorphic function $f(z)$ on Ω such that $f^{(j)}(z_k) = a_{kj}$ for $0 \leq j \leq m_k$ and for all k.

5.2. Suppose $\{u_n\}$ is a uniformly bounded sequence of harmonic functions on $\mathbb{D}$. Prove that there exists a subsequence $\{u_{n_\nu}\}$ that converges uniformly on compact subsets of $\mathbb{D}$.

5.3. Let $h(e^{i\theta})$ be a piecewise continuous function on the unit circle. Show that the Poisson integral $\tilde{h}(z)$ tends to $h(\zeta)$ as $z \in \mathbb{D}$ tends to any point ζ of the unit circle at which $h(e^{i\theta})$ is continuous.