

UWO Math 2124 Fall 2019
Introduction to Mathematical Problems
Instructor: Rasul Shafikov
Handout: Oct 1

Example 3.1: Consider finitely many points in the plane such that, if we choose any three points A, B, C among them, the area of triangle ABC always less than 1. Show that all of these points lie within the interior or on the boundary of a triangle with area less than 4.

Example 3.2: Several positive integers are written on a blackboard. One can erase any two distinct integers and write their *greatest common divisor* and *least common multiple* instead. Prove that eventually the numbers will stop changing.

Example 3.3: Given a unit square, show that if five points are placed anywhere inside or on this square, then two of them must be at most $\sqrt{2}/2$ units apart.

Example 3.4: Prove that from a set of ten distinct two-digit numbers (in the decimal system), it is possible to select two disjoint subsets whose members have the same sum.

Example 3.5: Let n be a positive integer. Choose any $(n+1)$ -element subset of $\{1, 2, \dots, 2n\}$. Show that this subset must contain two integers, one of which divides the other.