

Homework Assignment 1

Due Tuesday, October 1.

All work submitted must be your own; do not discuss this assignment with anyone except your course instructor. All solutions should be well-written and complete. A poorly written complete solution will not receive full credit. However, a well-written partial solution may receive substantial credit.

1. (5pt) Show that $n^2 + 1$ is not divisible by 4 for any positive integer n .
2. (i) (2pt) Given a binary (i.e., base 2) representation of an integer N , find a criterion of divisibility of N by 2.
(ii) (3pt) Given an integer represented base 7, find a divisibility rule by 6. Use your rule to determine if 163450311 (base 7) is divisible by 6.
3. (5pt) Let x be a real number such that $x + \frac{1}{x}$ is an integer. Prove that $x^n + \frac{1}{x^n}$ is an integer, for all positive integers n .
4. (5pt) A great circle is a circle drawn on a sphere that is an equator; i.e., its centre is also the centre of the sphere. There are n great circles on a sphere, no three of which meet at any point. They divide the sphere into how many regions?
5. (5pt) A merchant had a forty-pound measuring weight that broke into four pieces as the result of a fall. When the pieces were subsequently weighed, it was found that the weight of each piece was a whole number of pounds and that the four pieces could be used to weigh every integral weight between 1 and 40 pounds. What were the weights of the pieces?

Note: we can distinguish the two scales of the balance as the weight scale and the load scale. On the former we will place only pieces of the measuring weight; whereas on the load scale we will place the load and any additional measuring weights. If we are to make do with as few measuring weights as possible it will be necessary to place measuring weights on the load scale as well. For example, in order to weigh one pound with a two-pound and a three-pound piece, we place the two-pound piece on the load scale and the three-pound piece on the weight scale