

Homework Assignment 2

Due Tuesday, October 22.

All work submitted must be your own; do not discuss this assignment with anyone except your course instructor. All solutions should be well-written and complete. A poorly written complete solution will not receive full credit. However, a well-written partial solution may receive substantial credit.

1. (5pt) Assume that either $e < a < b$ or $0 < a < b < e$. Determine which number is bigger: a^b or b^a .
2. (5pt) You are given n points in the plane. You draw n circles, one centred at each of the n given points. Each of these circles is as large as it can be so that none of the given points, except its centre, are contained in its interior. Prove that there are (at least) two circles of the same size.
3. (5pt) You have 60 live pigeons and you give them to 10 kids, who stuff them into a 3×7 rectangular grid of 21 pigeonholes. They can stuff as many pigeons as they want into each hole. Show that you can always find 4 pigeonholes, positioned so that they form the corners of a rectangle, such that all 4 holes are empty, or all 4 have at least one pigeon in them.
4. (5pt) Prove that for any positive integer n , a $2^n \times 2^n$ square grid with any square removed can be covered with L-shaped (of size 2×1) tiles.
5. (5pt) Inside a 1×1 square, 101 points are placed. Show that some three of them form a triangle with area no more than 0.01.