**Homework Assignment 3**

Due Tuesday, November 12.

*All work submitted must be your own; do not discuss this assignment with anyone except your course instructor. All solutions should be well-written and complete. A poorly written complete solution will not receive full credit. However, a well-written partial solution may receive substantial credit.*

1. (5pt) All the chairs in a classroom are arranged in a square $n \times n$ array (in other words, $n$ columns and $n$ rows), and every chair is occupied by a student. The teacher decides to rearrange the students according to the following two rules: Every student must move to a new chair. A student can only move to an adjacent chair in the same row or to an adjacent chair in the same column. In other words, each student can move only one chair horizontally or vertically. (Note that the rules above allow two students in adjacent chairs to exchange places.) Show that this procedure can be done if $n$ is even, and cannot be done if $n$ is odd.

2. (5pt) Show that any positive integer containing exactly 2009 digits (in decimal representation), none of whose digits is zero, is either divisible by 2009, or can be changed to an integer that is divisible by 2009 by replacing some, but not all, of its digits by 0.

3. (5pt) Show that every graph contains two vertices of equal degree.

4. (5pt) Show that if a graph has $v$ vertices, each of degree at least $(v-1)/2$, then this graph is connected.

5. (5pt) A domino consists of two squares, each of which is marked with $0, 1, 2, 3, 4, 5$, or 6 dots. Verify that there are 28 different dominos. Is it possible to arrange them all in a circle so that the adjacent halves of neighbouring dominos show the same number?