Homework Assignment 4
Due Tuesday, December 3.

All work submitted must be your own; do not discuss this assignment with anyone except your course instructor. All solutions should be well-written and complete. A poorly written complete solution will not receive full credit. However, a well-written partial solution may receive substantial credit.

1. (5pt) How many strictly increasing sequences of positive integers begin with one and end with 1000?

2. (5pt) In how many ways can four squares, not all in the same row or column, be selected from an 8-by-8 chessboard to form a rectangle?

3. (5pt) Suppose m and n are positive integers. Prove that
$$\binom{m+n}{n} = \sum_{k=0}^{n} \binom{n}{k}\binom{m}{n-k}.$$

4. (5pt) How many ways can the positive integer n be written as an ordered sum of at least one positive integer? For example,
$$4 = 1+3 = 3+1 = 2+2 = 1+1+2 = 1+2+1 = 2+1+1 = 1+1+1+1,$$
So when $n = 4$ there are 8 such ordered partions.

5. (5pt) Prove that
$$n! < \left(\frac{n+1}{2}\right)^n, \text{ for } n = 1, 2, \ldots.$$