Problem Solving Session

Be prepared to discuss the following problems in class on Thursday, **December 5**.

1. The Fibonacci sequence is defined by $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n > 1$. So we have

$$\{f_n\} = \{0, 1, 1, 2, 3, 5, 8, 13, 21, \dots\}.$$

   Prove that for $n \geq 0$,

$$f_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right).$$

2. In how many ways can you make change for a dollar, using pennies, nickels, dimes, quarters, and half-dollars? For example 100 pennies is one way, 20 pennies + 2 nickels + 7 dimes is another. Order doesn't matter.