Problem Solving Session

Be prepared to discuss the following problems in class on Thursday, **October 17**.

1. Initially, we are given the sequence $1, 2, \ldots, 100$. Every minute, we erase any two numbers u and v and replace them with the value
$$uv + u + v.$$
Clearly, we will be left with just one number after 99 minutes. Does this number depend on the choices that we made?

2. Show that if every room in a house has an even number of doors, then the number of outside entrance doors must be even as well

3. Consider nine lattice points in three-dimensional space. Show that there must be a lattice point on the interior of one of the line segments joining two of these points. (A lattice in three-dimension space can be thought of as the set of all points in $\mathbb{R}^3$ with integer coordinates, e.g, $(-1, 2, 11)$.)